Name: ______________________________

MA162-020
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Quiz 7.5: Independence and conditional probability

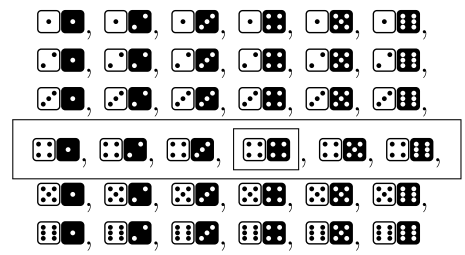
1. A survey of 10000 people conducted by the National Cancer Society reported that 3200 were "heavy coffee drinkers", 160 had cancer of the pancreas, and 132 were in both categories. Are the events of being a heavy coffee drinker and having cancer of the pancreas independent?

2. The probability that a battery will last 10 hours or more is 80%, but the probability that a battery will last 15 hours or more is only 15%. What is the probability that a battery will last 15 hours or more given that it has lasted 10 hours or more?

3. You need to show off in front of your boss. You've kept meticulous statistics of your dealings with customers, and you've noticed that 40% of female customers and 35% of male customers purchase something if you demonstrate a product nearby. There is a group of 5 females on one side of the store and a group of 3 females and 3 males on the other side of the store. Which side has the higher probability of buying something if you do a demonstration nearby? Should you bet on making a sale or try to find some other way of impressing your boss?

Examples 7.5: Independence and conditional probability

Suppose you roll two dice, and the first one comes up as a four. What is the probability the second one comes up as a four too?
Well there are only 6 ways the first dice can came up four, and in only one of those ways does the second dice also come up four, so that is 1 out of 6 or 17% chance.



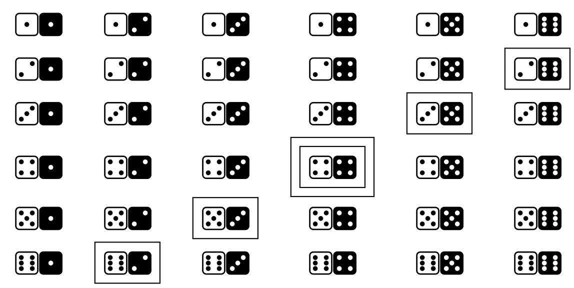
The **conditional probability** of the event A given the event B is defined to be:

$$P(A|B) = P(A \cap B)/P(B)$$

In the dice example we have B is the event the first die comes up four and A is the event the second die comes up four. The probability of both A and B individually is 1/6, but the probability of both A and B happening together, that is, the probability of $A \cap B$ is only 1/36 since we are saying exactly what happens: ⚃⚃. The conditional probability then is $(1/36)/(1/6) \approx 17\%$ just as above.

Suppose you roll two dice, and the sum is an eight. What is the probability the second die comes up as a four?

Well there are only 5 ways the sum can be an eight (⚁⚅, ⚂⚄, ⚃⚃, ⚄⚂, ⚅⚁), and only in one of those ways is the second die a four: ⚃⚃. That is 1 out of 5, or 20% chance.



Notice that the odds of getting an eight are 5/36 and the odds of the second die being a four are 6/36, but the conditional probability is 1/5, so the denominators can change a lot.

If we flip a coin once, there is a 50-50 chance of heads. If we flip a coin twice then there is a $(50\%)(50\%) = 25\%$ chance of getting heads twice. To get the probability of multiple events happening in independent trials, we just multiply probabilities (for the same reason we multiply when counting).

The last dice problem was different though: 5/36 chance of getting an eight (⚁⚅, ⚂⚄, ⚃⚃, ⚄⚂, ⚅⚁), and 6/36 chance of the second die being a four (⚀⚃, ⚁⚃, ⚂⚃, ⚃⚃, ⚄⚃, ⚅⚃), but there is $1/36 \approx 2.8\%$ chance of both happening (⚃⚃), not the product $(5/36)(6/36) \approx 2.3\%$.

Two events A and B are said to be **independent** if $P(A \cap B) = (P(A)) \cdot (P(B))$. Getting a heads on the first flip of a coin is independent of getting a heads on the second flip of a coin, but getting an eight on two dice is not independent of getting a four on the second die. We can rephrase the condition by dividing both sides by $P(B)$.

$$P(A|B) = P(A) \text{ if } A \text{ and } B \text{ are independent}$$

Is the probability of A given B the same as the probability of A? If so, then B has nothing to do with A and so A is independent of B.