

DEPARTMENT OF MATHEMATICS

Ma 162 Third Exam April 12, 2010

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations. You must show your work to receive credit.

Put your answers in the answer boxes provided, and show your work.

If your answer is not in the box or if you have no work to support your answer, you will receive no credit.

The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	16	
7	12	
8	12	
Total	100	

Please fill in the information below.

NAME: JACK SCHMIDT Section: 999
Last four digits of Student ID: 9999

Useful formulas.

1. Simple Interest: $I = Prt$. Accumulation: $A = P(1 + rt)$.
2. Compound Interest Accumulation: $A = P(1 + i)^n$. Present value: $P = A(1 + i)^{-n}$.
3. Effective rate: $r_{eff} = (1 + \frac{r}{m})^m - 1$.
4. Annuity: Sum: $S = R \frac{((1 + i)^n - 1)}{i}$. Present value: $P = R \frac{(1 - (1 + i)^{-n})}{i}$.
5. Set counting: Two sets: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ Three sets:
 $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

1. Suppose that P , Q and R are sets with 33, 38 and 37 members respectively. Calculate the indicated quantities. Display correct formulas or appropriate Venn diagrams.

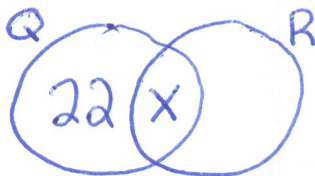
i) If $P \cap Q$ has 15 members, then $P \cup Q$ has 56 members.

$$\begin{array}{ccccccc}
 P \cup Q & & P & & Q & & P \cap Q \\
 X = & 33 & + & 38 & - & 15 & = & 56
 \end{array}$$

ii) If it is further known that $P \cap R$ has 13 members, then $P \cup R$ has 57 members.

$$\begin{array}{ccccccc}
 P \cup R & & P & & R & & P \cap R \\
 X = & 33 & + & 37 & - & 13 & = & 57
 \end{array}$$

iii) If, in addition, $Q - R$ has 22 members, then $Q \cap R$ has 16 members.



$$\begin{array}{ccc}
 Q - R & Q \cap R & Q \\
 22 + X & = & 38
 \end{array}$$

$$X = 38 - 22$$

iv) Finally, if we are given that the intersection of all three sets P , Q , and R has 7 members, then the union of all three sets has 71 members.

$$\begin{array}{ccccccccccc}
 P & & Q & & R & & P \cap Q & & P \cap R & & R \cap Q & & P \cap Q \cap R \\
 X = & 33 & + & 38 & + & 37 & - & 15 & - & 13 & - & 16 & + & 7
 \end{array}$$

A

2. Gary knows that he will need \$18,000 after about 3 years from now to buy new printers for his shop. He also notices that his current savings are not making much money and he decides to invest them in a growth fund to have enough money available after 3 years.

P just once

- a) How much would he have to invest now at 8% annual rate compounded annually to have that sum available?

\$14288.98

Be sure to show the formula used.

$A = 18000$	future dollars	$A = P(1+i)^n$
$i = 0.08$	per year	$P = A / (1+i)^n$
$n = 3$	years	$P = 18000 / (1+0.08)^3$
$P = ?$	present dollars	$P = 14288.98034$

- b) Suppose he finds a better investment opportunity which offers a bi-weekly compounding at the same annual rate. Assuming 26 nominal bi-weeks in the year,

how much would he need to invest in the new scheme?

\$14164.52

Be sure to show the formula used.

$A = 18000$	future dollars	$A = P(1+i)^n$
$i = 0.08/26$	per biweek	$P = 18000 / (1+0.08/26)^{78}$
$n = 78$	biweeks	$P = 14164.51990$
$P = ?$	present dollars	

money now

3. You are about to finance the purchase of a new house with a 20 year loan of 120 thousand dollars at 3.7% APR compounded monthly.

You are supposed to pay monthly. What is your monthly payment and what is your total of the payments over the period of 20 years?

Answer. Monthly Payment: $\$708.35$ dollars.

Total of payments: $\$170004.00$ dollars.

Be sure to show the formula used.

$$P = 120000$$

$$i = 0.037/12$$

$$n = 240$$

$$R = ?$$

present dollars

per month

months

dollars monthly

$$P = R \frac{(1 - (1+i)^{-n})}{i}$$

$$R = P / \left(\frac{(1 - (1+i)^{-n})}{i} \right)$$

$$R = 708.347429$$

Recurring Payment

$$T = Rn$$

$$(240 \text{ months})(708.35 \text{ per month})$$

$$T = (240)(708.35)$$

b) Suppose that another lender had offered the same loan with the same interest rate but for 30 years instead of 20 years. What will be the new monthly payment and the total of the payments over the period of 30 years?

Answer: Payment: $\$552.34$ dollars.

Total of payments: $\$198842.40$ dollars.

$$P = 120000$$

$$i = 0.037/12$$

$$n = 360$$

$$R = ?$$

$$R = P / \left(\frac{(1 - (1+i)^{-n})}{i} \right)$$

$$R = 552.339612$$

$$T = Rn$$

$$T = (552.34)(360)$$

4. John got a new job and is eager to replace his old car with a new one. But he does not want to take the responsibility of a new car loan. Therefore, he starts putting money into a savings account so he can buy a car after 4 years.

John figures that he would need to have \$15,000 saved, and he wishes to make monthly payments into an account paying 5% interest compounded monthly. Help him figure out the following.

Be sure to show the formulas used.

- a) How much should John's monthly payment be?

$\$282.94$

$A = 15000$ future dollars
 $i = 0.05/12$ per month
 $n = 48$ months
 $R = ?$ monthly dollars

$A = R((1+i)^n - 1) / (i)$
 $15000 = (R)(53.01489)$
 $R = 15000 / 53.01489$
 $R = 282.939780$

- b) At the end of 4 years, John decides to continue with his old car for another year yet continues making the same payments, so he can get a better car. How much

additional money will accumulate into the account?

$\$4241.64$

5 years total
 $A = ?$ 5 years dollars
 $i = 0.05/12$ per month
 $n = 60$ months
 $R = 282.94$ monthly dollars

$A = R((1+i)^n - 1) / (i)$
 $A = 19241.64281$ total
 but only 19241.64281
 $- 15000.00$

 4241.64281
 additional

5. In this problem, assume a 360 day year. "Payday!" will loan you 50% of your paycheck of \$600 for 10 days. After 10 days, you pay back the loan plus an interest of \$22.

a) Calculate the annual simple interest rate for this service. Answer:

264%

per cent.

$$P = \$300$$

$$r = i = ?$$

$$t = n = \frac{10}{360}$$

$$I = \$22$$

$$I = Prt$$

$$r = I / (Pt)$$

$$r = \cancel{300} / 22 / (300 (10/360))$$

$$r = \frac{66}{25} = 2.640000$$

- b) Across the street, "Cashit!" will loan you 5% more of your paycheck but charge you an interest of \$24. Is this a lower rate? Explain your answer by comparing the rates. Be sure to show all work.

$$P = \$330$$

$$r = ?$$

$$t = \frac{10}{360}$$

$$I = 24$$

$$I = Prt$$

$$r = 24 / (330 (10/360))$$

$$r = \frac{144}{55} = 2.618181818 \dots$$

$$r = \boxed{261.82\%}$$

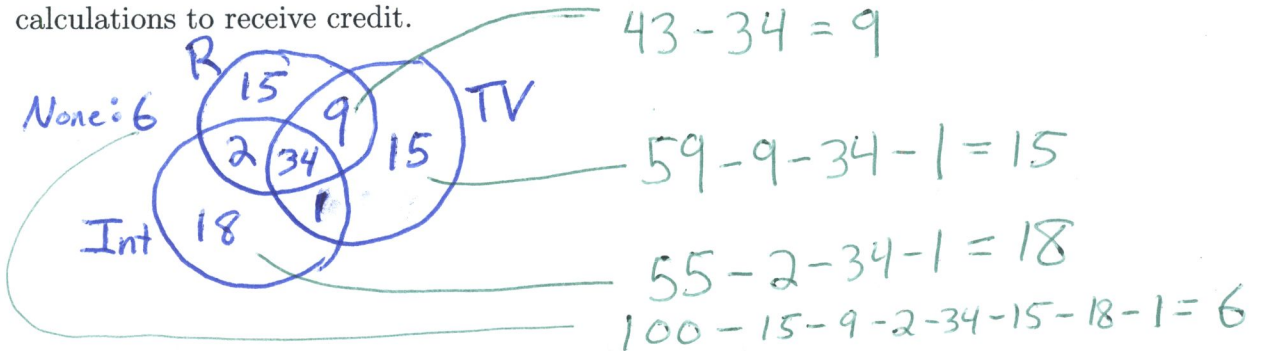
Yes this is a lower rate.

6. A survey of 100 College students revealed the following:

- 60 students use the Radio for their news.
- 59 students use TV for their news.
- 55 students use the Internet for their news.
- 43 students use both Radio and TV for their news.
- 36 students use Radio and Internet for their news.
- 35 students use TV and Internet for their news.
- 34 students use all three news sources.

} so only 1 uses TV and Int, but not Radio

Based on the above information, answer the following questions. You must show your calculations to receive credit.



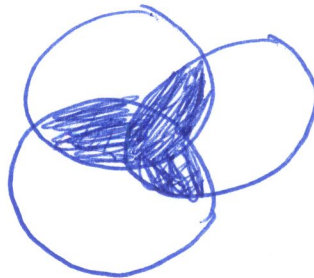
(a) How many students surveyed use none of the three news sources?

Answer:

(b) How many students surveyed use at least two of the three news sources?

Answer:

$2 + 34 + 9 + 1$



7. Nancy does not like to wear the same outfit twice. She has 4 different shirts, 7 different pants, and 6 different pairs of shoes.

a) How many successive days can she wear an outfit without repetition? **Answer:**

168

$$\begin{array}{ccc} \frac{4}{\text{Shirts}} & \frac{7}{\text{Pants}} & \frac{6}{\text{Shoes}} \end{array}$$

- b) Nancy goes out and buys two new shirts but unfortunately one of her shoes breaks down. Now, how many days can she go without repeating an outfit? **Answer**

210

$$\begin{array}{ccc} \frac{6}{\text{Shirts}} & \frac{7}{\text{Pants}} & \frac{5}{\text{Shoes}} \end{array}$$

8. There are 19 members in an executive committee. A subcommittee needs to be appointed for a special project. The subcommittee shall have a chairman, a public relations officer, a secretary and an accountant.

Answer the following questions. Be sure to show your reasoning. Just numerical answers shall earn no credit.

- (a) How many different subcommittees can be formed from the executive committee?

Answer: 93024 $\frac{19}{\text{Chair}} \frac{18}{\text{PR}} \frac{17}{\text{S}} \frac{16}{\text{A}}$
No Repeats Allowed

- (b) Suppose that the regular secretary of the executive committee has been assigned as the secretary of the sub committee. How many different subcommittees can now be formed?

Answer: 4896 $\frac{18}{\text{Chair}} \frac{17}{\text{PR}} \frac{1}{\text{S}} \frac{16}{\text{A}}$

1 Answer Key for exam3v-5

1.
 - ◇ (i) 56
 - ◇ (ii) 57
 - ◇ (iii) 16
 - ◇ (iv) 71

2.
 - ◇ a) \$14,288.98
 - ◇ b) \$14,164.52

3.
 - ◇ a) Monthly Payment \$ 708.35 Payout \$ 170,003.38
 - ◇ b) 552.34 Total 198,842.26

4.
 - ◇ a) Monthly saving \$282.94
 - ◇ Additional accumulation \$4,241.60

5.
 - ◇ a) 264.00 per cent
 - ◇ b) 261.82 per cent Better? yes!

6.
 - ◇ (a) 6
 - ◇ (b) 46

7.
 - ◇ a) 168 days
 - ◇ b) 210 days

8.
 - ◇ (a) 93024
 - ◇ (b) 4896