

DEPARTMENT OF MATHEMATICS

Ma 162 Final Exam -3 May 06, 2010

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure that your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations, but you must show your work to receive credit.

Put your answers in the answer boxes provided, and show your work.

If your answer is not in the box or if you have no work to support your answer, you will receive no credit.

The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	15	
2	15	
3	8	
4	10	
5	16	
6	12	
7	12	
8	12	
Total	100	

Please fill in the information below.

NAME: _____ Section: _____

Last four digits of Student ID: _____

1. Set this problem up, by stating the chosen variables, the function to be maximized and **all** the inequalities. **Do not solve the problem.**

“Trailblazers” company makes two types of trail mixes called “California” and “Tropical”.

Each 10 ounce pack of “California mix” sells for \$ 3.40, and uses 7.00 ounces of nut mix and 3.00 ounces of fruit mix.

Each 10 ounce pack of “Tropical mix” sells for \$ 2.80, and uses 4.00 ounces of nut mix and 6.00 ounces of fruit mix.

The company has 765 ounces of fruit mix on hand and 495 ounces of nut mix on hand.

Set up a LPP whose solution will determine how many packs of each type should be produced to maximize the company’s revenue.

- i) Define and explain all the variables you use.

Variables are:

- ii) Now describe the LPP explicitly by writing **all the inequalities** and specifying the objective function.

Maximize: Profit $P =$

Subject to:

- iii) The initial Simplex tableau will be:

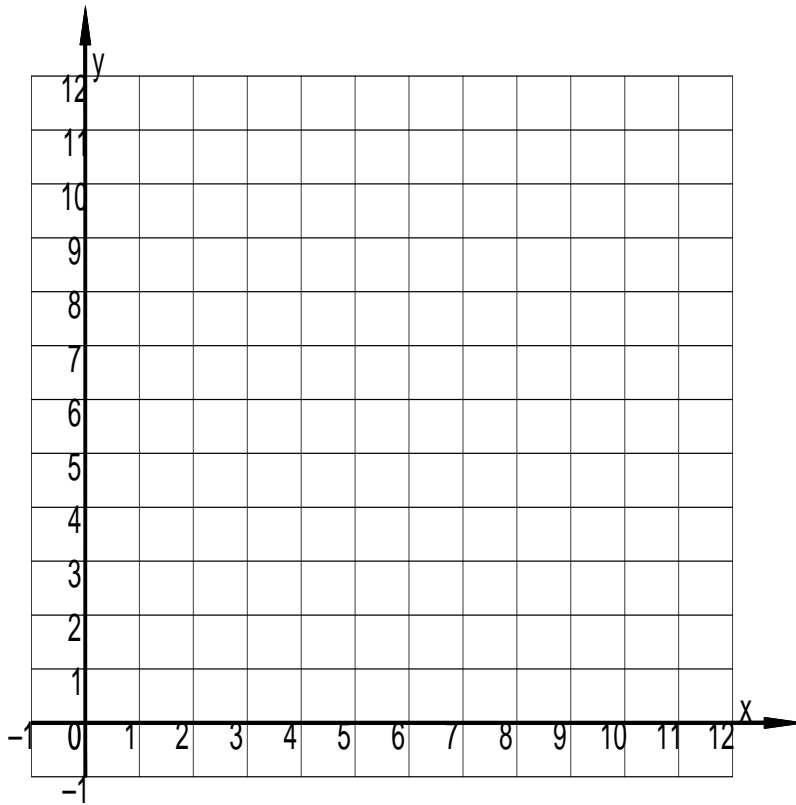
2. i) Sketch and shade the region described by the inequalities. Compute the coordinates of the corner points and mark them on your graph.

$$2x + 3y \leq 12$$

$$y \geq x - 5$$

$$x \geq 0$$

$$y \geq 0$$



- ii) Find the maximum value of the function, $P = 6x + 7y$ on the region.

Answer: $P = \boxed{}$ at $x = \boxed{}$, $y = \boxed{}$.

3. Here is a final tableau associated with a maximal LPP.

x	y	z	s	t	u	P	constants
-3	0	1	0	-5	-17	0	3
-1	1	0	0	0	1	0	4
1	0	0	1	1	4	0	2
5	0	0	0	4	21	1	13

(a) Using your knowledge of the Simplex algorithm, determine the solution to the maximal LPP.

Value of P = (x, y, z) = (, ,)

(b) Recall that the solution of the **dual minimization problem** can be read from the same tableau. Determine the solution of the dual minimization LPP.

Value of C = (s, t, u) = (, ,)

4. Set this problem up, by stating the chosen variables, the equations to be solved and **the initial augmented matrix. Do not solve the problem.**

Oliver has a farm with 240 acres. He is planning on planting sunflowers, wheat and corn and wants to decide how many acres to use for each crop.

Oliver's County agent Hank has advised him to plant as many acres of corn as acres of sunflowers and wheat together.

Oliver has a total of 12,540 dollars available for planting expenses and knows that sunflowers cost 32 dollars per acre, wheat costs 67 dollars per acre and corn costs 55 dollars per acre.

Oliver is wondering if it is possible to plant all the available acres while using up all the funds and following Hank's advice.

His wife Lisa tells him that all he has to do is to write and solve some linear equations.

Help Oliver by setting up equations for him and writing the initial augmented matrix.

Use x, y, z to denote the number of acres of "sunflowers", "wheat" and "corn" respectively.

The equations to be solved are:

The augmented matrix is:

5. (i) Consider the following system of linear equations.

$$x + 2y + z = 3$$

$$2x + 3y + 6z = 7$$

Write down the augmented matrix for this system of equations.

Reduce the augmented matrix to REF (the row echelon form). It is essential to show the steps of row reductions and explicitly write the row operations used. You may need more or fewer boxes than provided. Use the empty space on the back of the previous page if you need additional space.

- (ii) Using above calculations, determine all the solutions to the system of equations in x, y, z given above.

6. A group of 175 citizens were surveyed about their approval of the healthcare reform and the finance reform.

It was found that 91 citizens were for the healthcare reform and 76 were for the finance reform. In the survey 24 citizens announced that they did not approve of either reform and prefer to leave things as they are.

- (i) Estimate the probability that a random citizen likes some reform, i.e. he approves at least one of the two reforms.

- (ii) Estimate the probability that a random citizen approves both the reforms.

- (iii) Estimate the probability that a random citizen approves the healthcare reform but not the finance reform.

7. An experimenter has **an eight sided die** with numbers 1, 2, 3, 4, 5, 6, 7, 8 marked on its eight sides. He casts the die and records the number on the base of the die. Thus, his sample space has 8 samples points corresponding to the eight markings.

Let A be the event that the number on top is bigger than or equal to 4, i.e. one of 4, 5, 6, 7, 8.

Let B be the event that the number on top is less than or equal to 5, i.e. one of 1, 2, 3, 4, 5.

It is observed by experimentation that $P(A) = 55\%$ and $P(B) = 51\%$

Answer the following questions **based on these experimental observations.**

- (i) What is the probability that the number on bottom is 4 or 5 ? %.

- (ii) What is the probability that the number on bottom is 1 or 2 or 3 ? %.

- (iii) The experimenter concludes that this die must be loaded since a fair die would have the numbers 4, 5 on the bottom with probability equal to %.

8. Two fair dice are tossed, one red and one blue.

Let A be the event that the sum of the numbers on top is 5.

Let B be the event that the number on top of the red die is **not** 1. Answer the following:

(i) What is the probability that the sum of the numbers on top is 5, i.e. what is

$P(A)$?

(ii) the number on top of the red die is **not** 1, i.e. what is $P(B)$?

(iii) What is the probability that the sum of the numbers on top is 5, given that the number on top of the red die is **not** 1, i.e. what is $P(A|B)$?

(iv) What is $P(A \cap B)$?

(v) From the above calculations, would you consider A and B independent events? Why?

1 Answer Key for exam4v-3

1. $\diamond P = 3.4x + 2.8y \quad 0 \leq x \quad 0 \leq y \quad 3x + 6y \leq 765 \quad 7x + 4y \leq 495$

2.

$\diamond P = \frac{176}{5}$ at $x = \frac{27}{5} \quad y = 2/5$. $[[x = 5, y = 0], [x = \frac{27}{5}, y = 2/5], [x = 0, y = 4], [0, 0]]$

3. \diamond (a) $P = 13$ (x,y,z) $[0, 4, 3]$

\diamond (b) $C = 13$ (s,t,u) $[0, 4, 21]$

$$x + y + z = 240$$

4. $\diamond \quad 32x + 67y + 55z = 12540$

$$z = x + y$$

5. $\diamond \quad \begin{array}{cccccccc} 1 & 2 & 1 & 3 & 1 & 0 & 9 & 5 \\ 2 & 3 & 6 & 7 & 0 & 1 & -4 & -1 \end{array} \quad [x, y, z] = [-9z + 5, 4z - 1, z]$

6.

\diamond (i) $[\frac{151}{175}, "86.29", "percent"]$ (ii) $[\frac{16}{175}, "9.14", "percent"]$ (iii) $[3/7, "42.86", "percent"]$

7. \diamond (i) 6 % (ii) 45 % (iii) 25.0 %

8. \diamond (i) 0.1111 (ii) 0.8333 (iii) 0.1000 (iv) 0.08333000 (v) No $[0.09258, 0.08333]$