# MA162: Finite mathematics

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SCHEDULE:

- HW A1 is due Monday, Jan 24th, 2011.
- HW A2 is due Monday, Jan 31st, 2011.
- HW A3 is due Sunday, Feb 6th, 2011.
- Exam 1 is Monday, Feb 7th, 5:00pm-7:00pm.

Today we will cover:

- 1.3 linear functions; linear depreciation; cost, revenue, profit
- 1.4 intersections of lines; supply and demand

# Ch 1.3: Example 1: Linear depreciation

- In accounting, you keep track of assets (goods)
- But assets are also tax liabilities (bads)
- Old assets are like so whatever and are worth less
- For example:

  A printing machine is currently worth \$100,000, but will be depreciated over five years to its scrap value of \$30,000.

  How much is the machine worth after two years?

# Ch 1.3: Example 1: Linear depreciation

- For example:

  A printing machine is currently worth \$100,000, but will be depreciated over five years to its scrap value of \$30,000.

  How much is the machine worth after two years?

- Over five years, it loses \$70k of value
- Each year it loses $\$70\text{k}/5 = \$14\text{k}$ of value
- After two years, it loses $\$14\text{k} * 2 = \$28\text{k}$
- It is worth \$72k by the end of the second year

# Ch 1.3: Example 1: Linear depreciation

- This is just **slope**:
- $(x = 0, y = \$100k)$ and $(x = 5, y = \$30k)$
  are two points on the graph
- The slope is

$$\frac{100 - 30}{0 - 5} = -14 \text{ thousand dollars per year}$$

- The bunny hops down \$14k every year.
- The **y-intercept** was the original \$100k starting value

# Ch 1.3: Example 2: Cost, Revenue, Profit

- To get into the lucrative cell-phone washing business, you just need about \$5 in polishing rags and a winning smile
- However, each wash requires about \$0.05 in disinfectant
- If you charge \$0.25 per wash, how much money will you make if you wash 10 phones? 25 phones? 100?

# Ch 1.3: Example 2: Cost, Revenue, Profit

- Well your costs are easy: \$5 plus \$0.05 per wash

$$C(x) = 5 + 0.05x$$

- Your revenue is easy: \$0.25 per wash

$$R(x) = 0.25x$$

- So profit is easy, you start \$5 in the hole, and make \$0.20 per wash

$$P(x) = -5 + 0.20x$$

# Ch 1.3: Example 2: Cost, Revenue, Profit

- At 10 washes, you've made \$2.50 but spent \$5.50, so you are \$3 in debt
- At 25 washes, you've made \$6.25 but spent \$6.25, so you just broke even
- At 100 washes, you've made \$25 but spent \$10, so you are \$15 ahead

# Ch 1.3: Example 2: Cost, Revenue, Profit

- **Marginal cost** is \$0.05 per wash
- **Marginal profit** is \$0.20 per wash
- **Fixed cost** is \$5
- **Break-even production** is 25 washes

# Ch 1.4: Intersecting lines: Examples 2-5

- The break-even point is when the **revenue equals the cost**
- $R(x) = C(x)$
- To solve $0.25x = 5 + 0.05x$, move the $x$s over to get

$$0.20x = 5 \qquad x = 5/0.20 = 25$$

- A pessimistic phrasing is when the **profit is zero**
- $P(x) = 0$
- To solve $-5 + 0.20x = 0$, move the 5 over to get

$$0.20x = 5 \qquad x = 5/0.20 = 25$$

# Ch 1.3: Example 3: Demand function

- All else being equal, more people are willing to buy at a lower price
- Hopefully everyone took some graph paper
- Not very many people would take it if I charged $1 per sheet
- If 150 sheets are taken at $0 and none are taken at $1, about how many would be taken at $0.02?

# Ch 1.3: Example 3: Demand function

- With a **linear demand** model, this is easy:
- Every extra dollar I charge, I lose 150 customers
- If I only charge two extra pennies, I lose $150 * 0.02 = 3$ customers
- 147 pieces of paper should still circulate
- Real demand **curves** are not linear, but if the change in price is small enough, then they are like lines (remember MA123; curves look like lines close up; the derivative)

# Ch 1.3: Example 4: Supply function

- All else being equal, more are willing to sell if the price is higher
- If you heard Ovid's ran out of drinks and was paying \$20 per bottle of coke, some of you might leave class to make some money
- If no one is willing to supply coke for free, but 150 are willing to supply at \$100 per bottle, how many would be willing at \$20 per bottle?

# Ch 1.3: Example 4: Supply function

- All else being equal, more are willing to sell if the price is higher
- If you heard Ovid's ran out of drinks and was paying \$20 per bottle of coke, some of you might leave class to make some money
- If no one is willing to supply coke for free, but 150 are willing to supply at \$100 per bottle, how many would be willing at \$20 per bottle?
- By increasing the price \$100, we got 150 more sellers
- If we only increased the price a fifth of that, \$20, we would only get 30 more sellers

# Ch 1.4: Example 6-7: Market equilibrium

- In a rational, free market, the demand (number of items bought) equals the supply (number of items sold)
- On the exam, a problem like this requires you to:
  - find the supply equation
  - find the demand equation
  - set them equal to each other
  - solve for the **equilibrium quantity**
  - substitute back in for the **equilibrium price** (or vice versa)

# Ch 1.3 and 1.4 summary

- Concentrate on how the slope answers most of these questions with bunny hops
- There are also **tax** and **temperature** questions on the homework
- The homework and exams will use words like: **linear depreciation, cost function, revenue function, profit function, fixed costs, variable costs, supply equation, demand equation, market equilibrium**