MA162: Finite mathematics

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SOON:

- HW A1 is due Today, Jan 24th, 2011.
- I will be in the Mathskeller from 3pm-4pm today.
- HW A2 is due Monday, Jan 31st, 2011.
- HW A3 is due Sunday, Feb 6th, 2011.
- Exam 1 is Monday, Feb 7th, 5:00pm-7:00pm.

TODAY: Ch 2.1 Systems of linear equations

2.1: Systems of equations

- Supervising is hard. Workers get paid, but they need work to do
- You need to pick projects that keep them working
- It takes 15 minutes to assemble a MintyBoost,
  and 5 minutes of work to pack it
- It takes 10 minutes to assemble a TV-B-Gone,
  but 5 minutes of work to pack it
- Your assembly crew can work 40 hours per week,
  your packaging crew can work 15 hours per week
- So, how do you keep them busy?

2.1: Solving one with x and y

- We'll try one without words first:

$$\begin{cases} 4x + 3y = 10 \\ 7x - 2y = \ 3 \end{cases}$$

- Two variables, but two demands
- $x = 4$ and $y = -2$ works just fine in the first one:

$$4(4) + 3(-2) = 16 - 6 = 10$$

- But we aren't satisfied,

$$7(4) - 2(-2) = 28 + 4 = 32 \neq 3$$

- How can we make both equations true at the same time?

2.1: How to solve it? Try balance

- One way is to solve for one of the variables, and substitute back in
- In this case, none of the variables seems easy
- We can also "balance" the equations:
  multiply the top by 2 and the bottom by 3

$$\begin{cases} 4x + 3y = 10 \\ 7x - 2y = \;\;3 \end{cases} \xrightarrow[3\cdot\text{bot}]{2\cdot\text{top}} \begin{cases} \;\;8x + 6y = 20 \\ 21x - 6y = \;\;9 \end{cases}$$

- Now if we add 20 and 9 together, we get 29. Easy.
- But 20 and 9 have fancy names too, we can add them

2.1: Balanced equations become easier

- Let's add the fancy names too

$$\begin{cases} 8x + 6y = 20 \\ 21x - 6y = \ \ 9 \end{cases} \xrightarrow[\text{Add top to the bottom}]{\text{(leave it alone)}} \begin{cases} 8x + 6y = 20 \\ 29x + 0y = 29 \end{cases}$$

- But now the second equation is silly easy, $x = 1$

$$\begin{cases} 8x + 6y = 20 \\ 29x + 0y = 29 \end{cases} \xrightarrow[\text{Divide by 29}]{\text{(leave it alone)}} \begin{cases} 8x + 6y = 20 \\ x + 0y = \ \ 1 \end{cases}$$

- The first equation is easier now, since $x = 1$, $8x$ is just 8

$$\begin{cases} 8x + 6y = 20 \\ x + 0y = \ \ 1 \end{cases} \xrightarrow{\text{Subtract } 8\cdot\text{bot from top}} \begin{cases} 0x + 6y = 12 \\ x + 0y = \ \ 1 \end{cases}$$

2.1: Finishing up (backsolving)

- Finishing up by dividing $6y = 12$ by 6 to get $y = 2$

$$\begin{cases} 0x + 6y = 12 \\ x + 0y = 1 \end{cases} \xrightarrow{\text{Divide top by } 6} \begin{cases} 0x + y = 2 \\ x + 0y = 1 \end{cases}$$

- We don't need to write "0x" or "0y"

$$\begin{cases} 0x + y = 2 \\ x + 0y = 1 \end{cases} \xrightarrow{\hspace{6em}} \begin{cases} y = 2 \\ x = 1 \end{cases}$$

- Probably ought to check it worked: $(x = 1, y = 2)$

$$\begin{cases} 4x + 3y = 10 \\ 7x - 2y = 3 \end{cases} \xrightarrow{\text{Plug in}} \begin{cases} 4(1) + 3(2) = 4 + 6 = 10 \\ 7(1) - 2(2) = 7 - 4 = 3 \end{cases} \quad \checkmark$$

2.1: Example 1: Keeping the troops moving

- 15 minutes to assemble the MintyBoost, 5 minutes to pack it
- 10 minutes to assemble the TV-B-Gone, 5 minutes to pack it
- 40 hours is $40 \cdot 60 = 2400$ minutes of assembly
- 15 hours is $15 \cdot 60 = \ \ 900$ minutes of packing
- As a system of equations:

$$\begin{cases} 15x + 10y = 2400 & \text{assembly} \\ \ \ 5x + \ \ 5y = \ \ 900 & \text{packing} \end{cases}$$

- We finish x MintyBoosts and y TV-B-Gones

2.1: Example 1: Just the equations

- So we try to solve it:

$$\begin{cases} 15x + 10y = 2400 \\ 5x + 5y = 900 \end{cases}$$

$$\xrightarrow{\text{Subtract 2·bot from top}} \begin{cases} 5x + 0y = 600 \\ 5x + 5y = 900 \end{cases}$$

$$\xrightarrow{\text{Subtract top from bot}} \begin{cases} 5x + 0y = 600 \\ 0x + 5y = 300 \end{cases}$$

$$\xrightarrow{\text{Divide by 5}} \begin{cases} 1x + 0y = 120 \\ 0x + 1y = 60 \end{cases}$$

- So we ask our workers to make
  $x = 120$ MintyBoosts and
  $y = 60$ TV-B-Gones each week

2.1: Middle management adds more constraints

- Word comes down that you need to make twice as many MintyBoosts as TV-B-Gones
- They don't care about your need to keep the workers busy
- Now we must solve:

$$\begin{cases} 15x + 10y = 2400 \\ 5x + 5y = 900 \\ x - 2y = 0 \end{cases}$$

- Can we do it?
- You bet your minty boots we can!

2.1: Impossible mission

- Ok, what about this system:

$$\begin{cases} x + \ \ y = 4 \\ x + \ \ y = 8 \end{cases}$$

- People rarely are so direct about asking the impossible

$$\begin{cases} 2x + 2y = 8 \\ x + \ \ y = 8 \end{cases}$$

- But with two equations can the impossible really be disguised?

$$\begin{cases} 5x + 5y = 20 \\ 4x + 4y = 32 \end{cases}$$

2.1: The impossible game, k?

- Ok, so maybe if we have one equation and part of another, we can make it impossible

$$\begin{cases} x + y = 4 \\ 7x + ky = 56 \end{cases}$$

What value of k makes it impossible?

- Another:

$$\begin{cases} 3x - 7y = 13 \\ 6x + ky = 38 \end{cases}$$

What value of k makes it impossible?