MA162: Finite mathematics

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Schedule:

- Exam 1 is Today, Feb 7th, 5:00pm-7:00pm.
- HW B1 is due Monday, Feb 21st, 2011.

Today we will cover problems from the practice exam.

#1: Conversion problem

- $F = \frac{9}{5}C + 32$ when is F a third of C? $F = ?$

- Can $5F$ ever be $9C + 42$?

#1: Answer

- First part wants $F = (1/3)(C)$

- $(1/3)(C) = (9/5)C + 32$, so $((1/3) - (9/5))C = 32$, so $C = 32/((1/3) - (9/5)) = -240/11$

- $F = (1/3)(-240/11) = -80/11$

- The second part has nothing to do with first part

- The formula for F is $F = \frac{9}{5}C + 32$

- The formula for $5F$ is $5F = 9C + 160$

- $9C + 160 \neq 9C + 42$, so no.
  "Lines are parallel" or "No solution"

#2: Distance problem

- Dude goes from $A(0,0)$ to $C(12,10)$, passing through both $B(7,5)$ and $D(5,7)$. How does he get there the quickest?

$AB =$

$BD =$

$DC =$

$AD =$

$DB =$

$BC =$

#2: Answer

- From $A(0,0)$ to $C(12,10)$, passing through $B(7,5)$ and $D(5,7)$.

$$AB = \sqrt{7^2 + 5^2} = \sqrt{74}$$

$$BD = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$DC = \sqrt{7^2 + 3^2} = \sqrt{58}$$

$$AD = \sqrt{5^2 + 7^2} = \sqrt{74}$$

$$DB = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$BC = \sqrt{5^2 + 5^2} = \sqrt{50}$$

- Shortest is latter, ADBC, through D first, total distance is $\sqrt{74} + \sqrt{8} + \sqrt{50} \approx 18.5$

#3: Points and slopes and triangles

- Distance from $A(3,1)$ to $B(10,0)$?

- Slope from $A(3,1)$ to $B(10,0)$?

- Point $C(4,y)$ forms a right triangle with $A(3,1)$ and $B(10,0)$ with the right angle at A?

#3: Answer to first parts

- Problem: Find distance from $A(3,1)$ to $B(10,0)$
- Triangle solution: the horizontal side is length $10-3=7$, the vertical side is length $1-0=1$, so the distance is

$$d_{AB} = \sqrt{7^2+1^2} = \sqrt{50}$$

- Formula solution:

$$d_{AB} = \sqrt{(3-10)^2+(1-0)^2} = \sqrt{49+1} = \sqrt{50}$$

- Problem: Find slope from $A(3,1)$ to $B(10,0)$
- Down 1, over 7, so slope is:

$$m_{AB} = \frac{0-1}{10-3} = \frac{-1}{7}$$

#3: Answer to third part

- Problem: The points A(3,1), B(10,0), and C(4,y) form a right triangle with right angle at A. Find y.
- Idea of solution: Since BAC is a right angle, BA and AC are perpendicular, so we can use slope to find y.
- The slope of BA is $(0-1)/(10-3) = (-1)/(7)$, so the slope of AC is $(+7)/(1)$, the "opposite reciprocal".
- Bunny hop finish: To get from A to C is one hop to the right, so 7 hops up. $y = 1 + 7 = 8$.
- Algebra finish: The equation of the line through AC is $y = 7x + b$. Plugging in $A(x = 3, y = 1)$ we get $1 = (7)(3) + b$, so $b = -20$, and in general $y = 7x - 20$. Plugging in $C(x = 4, y = ?)$ we get $y = (7)(4) - 20 = 8$.

#4: Cost, Revenue, Profit

- Cost is $C = 4x + 6300$, marginal revenue is 11, what is profit function?

- What is break-even value and cost?

#4: Answer

- $R = 11x$, $P = R - C = 11x - (4x + 6300) = 7x - 6300$

  \$7 marginal profit, \$6300 fixed cost

- Break even when $7x = 6300$, when $x = 900$
- Break even cost is $(4)(900) + 6300$
- Same as break even revenue of $(11)(900) = 9900$

- If your answer comes out as a fraction, leave it as a fraction is ok for exam

#5: Supply and demand

- Supply obeys $x = 40p + 100$, demand is linear through $(p = \$1, d = 540)$ and $(p = \$10, d = 0)$. What is the equilibrium price and quanity?

- Step 1: What is the demand equation?

- Step 2: Intersect them

#5: Answer for step 1

- $d = Ap + B$ for some numbers A and B
- Plug in $(p = \$1, d = 540)$ to get
$$540 = A + B$$
- Plug in $(p = \$10, d = 0)$ to get
$$0 = 10A + B$$
- Subtract to get
$$540 = -9A \qquad A = -60$$
$$540 = -60 + B \qquad B = 600$$
$$d = -60p + 600$$

#5: Answer for step 2

$$\begin{cases} x = 40p + 400 \\ d = -60p + 600 \end{cases} \qquad \text{Equilibrium means: } x = d$$

$$40p + 400 = -60p + 600 \qquad 100p = 200 \qquad p = 2$$

$$x = (40)(2) + 400 = 480$$

$$d = (-60)(2) + 600 = 480$$

- $x = d$, yay!

#6: The k-game

- What is k when this system has no solution?

$$\begin{cases} x - 2y + z = 1 \\ 2x + y + 3z = 0 \\ y + kz = 0 \end{cases}$$

#6: Answer

- Write it as matrix, then REF:

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & k & 0 \end{array}\right) \xrightarrow{R_2-2R_1} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & 1 & -2 \\ 0 & 1 & k & 0 \end{array}\right) \xrightarrow{R_2\leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & k & 0 \\ 0 & 5 & 1 & -2 \end{array}\right)$$

$$\xrightarrow{R_3-5R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1-5k & -2 \end{array}\right)$$

- Pivots in each variable's column unless $1 - 5k = 0$
- so k must be $\frac{1}{5}$

#7: REF it

$$\begin{cases} -x + y + 3z = 0 \\ 2x - y - 4z = -1 \\ 2x - 2y - 5z = 2 \end{cases}$$

- Write it as an augmented matrix.
- Use standard row operations to bring it to REF
  (show work for sure)

#7: Answer

$$\left(\begin{array}{rrr|r} -1 & 1 & 3 & 0 \\ 2 & -1 & -4 & -1 \\ 2 & -2 & -5 & 2 \end{array}\right) \xrightarrow[R_3+2R_1]{R_2+2R_1} \left(\begin{array}{rrr|r} -1 & 1 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) \text{REF!}$$

#8: Read the REF answer

$$\left\{\begin{array}{rcrcrcr} -x & + & y & - & 3z & = & -3 \\ 2x & - & y & + & 5z & = & 5 \\ 2x & - & 2y & + & 7z & = & 8 \end{array}\right. \to \cdots \to \begin{pmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

- How many solutions?
- $x =$
- $y =$
- $z =$

#8: Answer

- $x + 2z = 2$, so
$$x = 2 - 2z$$
- $y - z = -1$, so
$$y = z - 1$$
- $z = 2$, so
$$z = 2$$
$$y = 2 - 1 = 1$$
$$x = 2 - 4 = -2$$
- One solution: $(x = -2, y = 1, z = 2)$