

# MA162: Finite mathematics

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## SCHEDULE:

- HW B1 is due Today, Feb 21st, 2011.
- HW B2 is due Monday, Feb 28th, 2011.
- HW B3 is due Sunday, Mar 6th, 2011.
- Exam 2 is Monday, Mar 7th, 5:00pm-7:00pm.
- Alternate request form on [MathClass.org](http://MathClass.org):  
Due Wed Feb 23 (sorry for short notice).

Today we will cover 3.2: Linear programming problems

## Exam 2: Overview

- 35% Ch 2, Matrix arithmetic (HWB1):
  - Sizes, Addition, subtraction, multiplication by a number, matrix-matrix multiplication, matrix to a power
  - Matrix inverse ( $2 \times 2$  and  $3 \times 3$ ); Solving system given inverse
- 30% Ch 3, Two-variable linear programs (HWB2):
  - **Setting up a linear programming problem**
  - Solving using the graphical method
- 35% Ch 4, Simplex algorithm (HWB3)

## 3.2: Linear programming problems

- An LPP has three parts:
  - The variables (the business decision to be made)
  - The inequalities (the laws, constraints, rules, and regulations)
  - The objective (maximize profit, minimize cost)
- Setting up the problem will be **your job!**
- Reading the answer will be **your job!**
- The middle part is on the exam and you can do it!

## 3.2: Example 1. Production problem (1/2)

- Ace Novelty is a small company producing two products:
  - Monogrammed water bottles with custom cozy
  - Ornamental sphere and reptile pack (OSARP)
- It uses modern micro-manufacturing techniques including its:
  - MakerBot computer aided 3D printer
  - KnitBot-2010 computer controlled knitting machine
  - Assembly crew (people)

## 3.2: Example 1. Production problem (2/2)

- Each Water bottle realizes the company a profit of \$10  
Each OSARP realizes the company a profit of \$12
- Each item requires a certain amount of time (in minutes):

	3D Printer	KnitBot	Crew
Bottle	26	60	20
OSARP	62	30	40

- Time is short: Each day the company can only run the 3D printer 5 hours, the KnitBot 4 hours, and the crew 4 hours.
- The union is strong: The total machine time can only be three times as much as the human time
- How can you maximize profit without destroying the machines or ticking off the union?

## 3.2: Example 1. Setting it up (1/3)

- What do you actually have control over?
  - Can you buy better machines?
  - Can you bribe the union leader?
  - Can you make time STAND STILL?!
- Maybe you should start by deciding how many bottles and how many OSARPs to make.
- The manager (you) sets the **Production Goals** in order to maximize profit legally
- We use **variables** to describe our decision:
  - $X$  = the number of water bottles to make each day
  - $Y$  = the number of OSARPs to make each day

## 3.2: Example 1. Setting it up (2/3)

- What constraints do we operate under?

$$26X + 62Y \leq 300 \quad (\text{3D printer time})$$

$$60X + 30Y \leq 240 \quad (\text{KnitBot time})$$

$$20X + 40Y \leq 240 \quad (\text{Human time})$$

$$26X - 28Y \leq 0 \quad (\text{Union req.})$$

- Sanity:  $X \geq 0$ ,  $Y \geq 0$  (standard inequalities)

- Union requirement:

Machine time is  $26X + 60X + 62Y + 30Y = 86X + 92Y$  and

Human time times three is  $3(20X + 40Y) = 60X + 120Y$

So requirement is  $86X + 92Y \leq 60X + 120Y$ , or

$$26X - 28Y \leq 0$$

## 3.2: Example 1. Setting it up (3/3)

- Ok, no problem. I have the answer.  $X = 0$  and  $Y = 0$ . No rules are broken!
  - We need a **goal**. We need an **objective**:
  - **Maximize** the profit  $P = 10X + 12Y$
- 
- We can do a lot better than  $X = 0$  and  $Y = 0$  (with  $P = 0$ )
  - Even  $X = 1$  and  $Y = 1$  is better! ( $P = 22$  and no rules broken)

## 3.2: Example 1. Summary

- **Variables:**

$X$  = the number of water bottles to make each day

$Y$  = the number of OSARPs to make each day

- **Constraints:**

$$26X + 62Y \leq 300 \quad (\text{3D printer time})$$

$$60X + 30Y \leq 240 \quad (\text{KnitBot time})$$

$$20X + 40Y \leq 240 \quad (\text{Human time})$$

$$26X - 28Y \leq 0 \quad (\text{Union req.})$$

and  $X \geq 0, Y \geq 0$

- **Objective:**

Maximize the profit  $P = 10X + 12Y$

- (Done! We just want to set the problem up!)

## 3.2: Example 2. Nutrition

- A Food-and-Nutrition-Science student was asked to design a diet for someone with iron and vitamin B deficiencies
- The student said the person should get at least 2400mg of iron, 2100mg of vitamin  $B_1$ , and 1500mg of vitamin  $B_2$  (over 90 days)
- The student recommended two brands of vitamins:

	Brand A	Brand B	Min. Req
Iron	40mg	10mg	2400mg
$B_1$	10mg	15mg	2100mg
$B_2$	5mg	15mg	1500mg
Cost:	\$0.06	\$0.08	

- The client asked the student to recommend the **cheapest** solution
- How many pills of each brand should the person get in order to meet the nutritional requirements at the minimal cost?

## 3.2: Example 2. Setting it up

- **Variables:**

$X$  = number of pills of brand A

$Y$  = number of pills of brand B

- **Constraints:**

$$40X + 10Y \geq 2400 \quad (\text{Iron})$$

$$10X + 15Y \geq 2100 \quad (\text{B1})$$

$$5X + 15Y \geq 1500 \quad (\text{B2})$$

and  $X \geq 0, Y \geq 0$

- **Objective:**

Minimize cost  $C = 0.06X + 0.08Y$

## 3.2: Example 3. Shipping costs

- You hit the big time, Mr. or Ms. Big Shot.  
You've got two manufacturing plants and two assembly plants
- Your assembly plants A1 and A2 need 80 and 70 engines
- Your production plants can produce up to 100 and 110 engines
- The shipping costs are:

From	To assembly plant	
	A1	A2
P1	100	60
P2	120	70

- How many engines should each production plant ship to each assembly plant to meet the production goals at the minimum shipping cost?

## 3.2: Example 3. Setting it up (1/3)

- What do you have control over? Four things?

$X$  = Number of engines from P1 to A1

$Y$  = Number of engines from P1 to A2

$Z$  = Number of engines from P2 to A1

$\xi$  = Number of engines from P2 to A2

- But do we really need all these variables?

How many engines does A1 even want?

- $X + Z = 80$  and  $Y + \xi = 70$

- Why not just use  $X$  and  $Y$ ?

$Z$  and  $\xi$  are just “the rest”

## 3.2: Example 3. Setting it up (2/3)

- What are the requirements?
- Sanity is complicated:  $X \geq 0$ ,  $Y \geq 0$ ,  $Z \geq 0$ ,  $\xi \geq 0$
- But wait, we got rid of  $Z$  and  $\xi$ !  
No big deal, just don't ship more than needed!
- Sanity:  $0 \leq X \leq 80$  and  $0 \leq Y \leq 70$
- Only other constraint is production capacity:
- $X + Y \leq 100$  from P1 capacity
- $Z + \xi \leq 110$  from P2 capacity
- Rewrite P2 as  $(80 - X) + (70 - Y) \leq 110$  really just  $40 \leq X + Y$

## 3.2: Example 3. Setting it up (3/3)

- What is the goal?
- Cost is complicated:  $100X + 60Y + 120Z + 70\xi$
- Rewrite as  $100X + 60Y + 120(80 - X) + 70(70 - Y)$
- Simplifies to  $C = 9600 - 20X + 4900 - 10Y = 14500 - 20X - 10Y$
- Ok, but we need an executive summary, this was too long!

## 3.2: Example 3. Summary

- **Variables:**

$X$  = Number of engines from P1 to A1

$Y$  = Number of engines from P1 to A2

$80 - X$  = Number of engines from P2 to A1 (the rest of A1's demand)

$70 - Y$  = Number of engines from P2 to A2 (the rest of A2's demand)

- **Constraints:**

$$X + Y \leq 100 \quad (\text{P1 max production})$$

$$X + Y \geq 40 \quad (\text{P2 max production})$$

$$X \leq 80 \quad (\text{sanity, A1 max demand})$$

$$Y \leq 70 \quad (\text{sanity, A2 max demand})$$

and  $X \geq 0, Y \geq 0$

- **Objective:**

minimize shipping cost  $C = 14500 - 20X - 10Y$

## 3.2: Example 4. Fancy shipping

- Two plants P1 and P2 and three warehouses W1, W2, W3
- Shipping costs are in the following table:

	W1	W2	W3
P1	20	8	10
P2	12	22	18

- Maximum production and minimum requirements are:

P1	400
P2	600

	W1	W2	W3
Cost	200	300	400

## 3.2: Example 4. Setting it up (1/3)

- We honestly have six variables! We'd run out of letters.
- $X_1, X_2, X_3, X_4, X_5, X_6$  are six different variables
- They are pronounced "Ecks One, Ecks Two, Ecks Three, ..."
- The number is just like a serial number, it doesn't mean multiply or square or anything like that
- So our variables are:  $X_1 =$  number to ship from P1 to W1  
 $X_2 =$  number to ship from P1 to W2  
 $X_3 =$  number to ship from P1 to W3  
 $X_4 =$  number to ship from P2 to W1  
 $X_5 =$  number to ship from P2 to W2  
 $X_6 =$  number to ship from P2 to W3

## 3.2: Example 4. Setting it up (2 and 3/3)

- What are the constraints?

Max production, and min reception

$$x_1 + x_2 + x_3 \leq 400 \quad (\text{P1 max prod})$$

$$x_4 + x_5 + x_6 \leq 600 \quad (\text{P2 max prod})$$

$$x_1 + x_4 \geq 200 \quad (\text{W1 min supply})$$

$$x_2 + x_5 \geq 300 \quad (\text{W2 min supply})$$

$$x_3 + x_6 \geq 400 \quad (\text{W3 min supply})$$

and  $x_1 \geq 0$ ,  $x_2 \geq 0$ ,  $x_3 \geq 0$ ,  $x_4 \geq 0$ ,  $x_5 \geq 0$ , and  $x_6 \geq 0$ .

- What is the objective?

Minimize cost:  $C = 20x_1 + 8x_2 + 10x_3 + 12x_4 + 22x_5 + 18x_6$