MA162: Finite mathematics

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University of Kentucky

April 20, 2011

SCHEDULE:

- HW D2 is due Monday, Apr 25th, 2011.
- HW D3 is due Friday, Apr 29, 2011.
- Final Exam is Wednesday, May 4th, 6:00pm-8:00pm.
- There is an alternate signup sheet due Thursday, April 21st

Today we will cover 7.3: Rules of probability

Final exam breakdown

- Chapter 1 and 2: Linear systems:
  - Convert a word problem to a system of equations
  - Convert a system of equations to matrix,
    REF or RREF it,
    backsolve or read solution,
    "Free variables"
- Chapter 3 and 4: Linear optimization:
  - Convert a word problem to a system of inequalities
  - Solve a system of inequalities using the graphical method
  - Read a solution from the final tableau of a simlex algorithm
- Chapter 6 and 7: Counting and probability:
  - Inclusion-exclusion in probability
  - Fair gambling
  - Unfair?

7.2: Just count for probability

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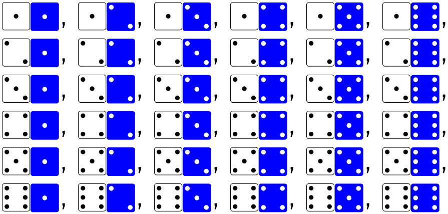

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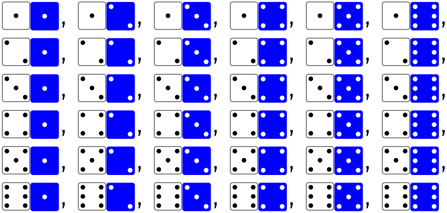

- The second row and the fifth column work: $P = \frac{6+6-1}{(6)(6)} = \frac{11}{36}$

7.2: Crazy counting

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- What is the probability of getting at least 2 aces out of 3 cards?
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$$P(\text{at least } 2) = \frac{C(4,2)C(20,1) + C(4,3)}{C(24,3)} = \frac{30}{506} + \frac{1}{506} = \frac{31}{506}$$

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- $P(E - F) = P(E) - P(E \cap F) = 40\% - 10\% = 30\%$

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- $Pr(E) = Pr(E \cap F) + Pr(E - F)$
- Every counting problem formula you can imagine has a probability counterpart

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- $1 - (1 - \frac{1}{6})^3$ chance of THAT not happening

$$\frac{91}{216} = 1 - \left(1 - \frac{1}{6}\right)^3$$

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