DEPARTMENT OF MATHEMATICS

Ma 162 First Exam September 26, 2011

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	**Maximum Score**	**Actual Score**
1	10	
2	10	
3	10	
4	15	
5	10	
6	15	
7	10	
8	10	
9	10	
Total	100	

NAME: ______________________________ **Section:** ____________

Last four digits of Student ID: __________________

1. A courier travels from city Ashton with coordinates $(0, 0)$ to city Cranston with coordinates $(76, 41)$. He must pass through **exactly one of the cities** Brady with coordinates $(16, 30)$ or Dalton $(48, 20)$ along the way. Assume he travels a straight line between cities.

   (a) Which city should he pass through (Brady or Dalton) in order to minimize his trip distance from Ashton to Cranston?

   He should pass through city [   ] on his way to Cranston.

   (b) What is the total minimum length of his trip from Ashton to Cranston, taking into account the stop in the city from part (a)?

   Minimum trip length is: [   ]

2. Point A has coordinates $(7, 3)$,and point B has coordinates $(2, 5)$.

(a) What is the distance from A to B and what is the slope of the line joining A to B?

distance: [ ], slope: [ ]

(b) Suppose that the point C with coordinates $(x, 10)$ is such that the triangle ABC is a right triangle with right angle at B. Determine the value of x. (Note: The coordinates of $A(7, 3)$ and $B(2, 5)$ were given at the top of the problem.)

$x =$ [ ]

3. The Flörgerstrøm company makes valve cleaning units for flügelhorns. The cost function for their manufacturing line is $C = 3x + 4300$, where x is the number of VCUs produced per month and C is measured in dollars. The company expects \$13 in revenue per unit.

   (a) Determine the linear profit function for the Flörgerstrøm company in the usual form $P = mx + b$, assuming they can sell all the units they manufacture.

   $P =$

   (b) Determine the break-even value for x and the break-even cost C at that value for x.

   $x =$ $C =$

4. In a free market, the supply equation for a supplier of corn is $x = 36p + 200$ where the price p is in dollars and x is in bushels. When the price is \$4 per bushel the demand is 1170 bushels. When the price goes up to \$17 per bushel the demand drops to 0 bushels. Assuming that the demand equation is also linear, find the equilibrium price and the number of bushels supplied at that equilibrium price.

Demand equation: [ ]

$p =$ [ ] $x =$ [ ] bushels

5. For the following word problem: (a) Write down variables describing the (numerical) business decision to be made, (b) write down equations that constrain your decision, (c) convert the equations to an augmented matrix. **You need not solve the system.**

   Mrs. Oregano runs a spaghetti storehouse, but wants to clear it out before she goes out of business. She has a rather large inventory of fabulous flour, tasty tomatoes, and glorious garlic. She decides she is going to use every last bit of her inventory to make the 2011 Oregano Outdoor Feast! Her feast only includes Spaghetti, Bruschetta, and Ziti. Flour is used in all three dishes: 1/8 cup per Spaghetti dinner, 3/4 cup per Bruschetta plate, and 1/4 cup per Ziti dinner. Tomatoes get used in all three dishes too: 1 cup per Spaghetti dinner, 1/2 cup per Bruschetta plate, and 1 cup per Ziti dinner. Actually garlic gets used in all three dishes too: 2 cloves per Spaghetti dinner, 1 clove per Bruschetta plate, and 1.5 cloves per Ziti dinner. Mrs. Oregano has 80 cups of flour, 160 cups of tomatoes, and 300 cloves of garlic. How many dishes of each type should she make in order to use up all of her inventory?

The variables describing the decision are:

The equations to be solved are:

The augmented matrix describing the equations is:

6. Here is the augmented matrix of a linear system of equations. Take this matrix to RREF. Be sure to label your reduction operations in standard notation. You need not solve for the variables.

$$\left(\begin{array}{cccc|c} x & y & z & w & \text{RHS} \\ \hline 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 2 & 2 \end{array}\right)$$

7. Here is the augmented matrix of a linear system of equations. As usual, the variables are mentioned for your convenience.

$$\left(\begin{array}{cccc|c} x & y & z & w & \text{RHS} \\ \hline 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$

(a) Is this matrix in REF or RREF or neither of these?

(b) Finish the solution process as needed and determine the complete solution of the system by filling in the answers below. If a variable is free, then enter the word "free" as its value. Be sure to show all calculations.

$x =$ [ ]

$y =$ [ ]

$z =$ [ ]

$w =$ [ ]

8. Consider the following matrices and answer the questions. In each case, either calculate the expression or explain why it is not defined.

$$Q = \begin{bmatrix} 4 & 5 & 6 \\ -4 & -5 & -6 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad S = \begin{bmatrix} 1 & -4 \\ -4 & -2 \end{bmatrix} \quad T = \begin{bmatrix} 3 & 3 \\ 3 & 3 \\ 3 & 3 \end{bmatrix}$$

(a) $Q + R$

(b) $2T - R$

(c) QR

(d) ST

(e) $S^2 - 3S$

9. (a) Find the inverse of the matrix $P = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$.

Answer:

(b) Suppose that $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$.

Determine the solution X of the equation $AX = B$.

Answer: