MA162: Finite mathematics

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SCHEDULE:

- Exam 3 is Monday, Nov 14th, 5:00pm-7:00pm in CB106.

Today we will cover 5.1: simple and compound interest.
We will be using calculators today.

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations

5.1: Interest

- Businesses often need short-term use of expensive assets, so find renting attractive (often tax-deductible)

- Sometimes what a business needs most is just cash.
  In a small business, you don't make money every day.
  A successful small business does make money,
  so can repay the money in the future.

- How can they rent cash?
  Why would somebody give them money today?
  For the promise of more money in the future. **Interest**

- How much more?

  - The more money being loaned, the more interest. **Principal**

  - The longer the money is loaned, the more interest. **Time**

5.1: Simple interest

- For short term loans, people use a **simple** model for interest

$$I = Prt$$

- There is the **Principal**, the amount of money borrowed, like \$100
- There is a **rate** of interest, like 10% per year
- There is a **time** period, after which the money is due, like 1 year
- There is the **Interest**, the extra money due at the end,

  like $(\$100) \cdot (10\% \text{ per year}) \cdot (1 \text{ year}) = \$10$.

5.1: Consumer example

- My Brother-in-Law's electricity bill came too soon one month
- Bill was \$46.40 now, but \$48.72 if 3 days late
- He didn't have the money now, but would have it in a week (IRS refund)
- He did have a 48% APR credit card carrying a balance (4% interest per month)
- A pawn shop would loan him the money for one month
  2% interest per month, \$5 fee
- Which is cheaper:
  - (L) Pay it late
  - (R) Put it on the credit card, and pay the credit card
  - (B) Pawn his watch for a month, then pay it back

5.1: Let's just see how much each costs

- (L) is easy: \$48.72 total, \$2.32 in interest
- (R) is easy: \$46.40 plus 4% $=$ \$46.40(1.04) $=$ \$48.26
- (B) is easy: \$46.40 plus 2% plus \$5 $=$ \$46.40(1.02) $+ 5 =$ \$52.33
- Decision is also easy: credit card is the cheapest
- If he had the money now, then cheapest was to pay it now \$46.60
- There is a price to not having money

5.1: Simple interest examples

$$I = Prt$$

- If \$100 is lent at 10% interest per year for six months, then

$$I = (\$100) \cdot (10\% \text{ per year}) \cdot (\tfrac{1}{2} \text{ year}) = \$5$$

- If \$100 is lent at 7% interest per year for three months, then

$$I = (\$100) \cdot (7\% \text{ per year}) \cdot (\tfrac{1}{4} \text{ year}) = \$1.75$$

- If \$325 is lent at 12% interest per year for five months, then

$$I = (\$325) \cdot (12\% \text{ per year}) \cdot (\tfrac{5}{12} \text{ year}) = \$16.25$$

5.1: More examples

- What is the simple (yearly) interest rate if \$100 is loaned for 3 months with \$5 interest due?

$$\begin{aligned} P &= \$100 \\ t &= 1/4 \text{ year} \\ I &= \$5 \\ r &= ? \end{aligned} \qquad \begin{aligned} I &= Prt \\ 5 &= (100)(r)(1/4) \\ 5 &= 25r \\ r &= 1/5 = 20\% \end{aligned}$$

- If the interest rate is 7% and \$9.10 interest is due after three months, how much was loaned?

$$\begin{aligned} r &= 7\% \text{ per year} \\ t &= 1/4 \text{ year} \\ I &= \$9.10 \\ P &= ? \end{aligned} \qquad \begin{aligned} I &= Prt \\ \$9.1 &= P(7\%)(1/4) \\ \$36.4 &= P(7\%) \\ P &= \$36.4/7\% = \$520 \end{aligned}$$

5.1: Simple interest as rent

- Simple interest treats loaned money like a loaned house.
- Every month you pay money to borrow the house, and at the end of the year you give the house back.
- The owner goes without his house for a year, but receives money in exchange (rent). The lender goes without his money for a year, but receives money in exchange (interest).
- How much rent do you pay total? If each month you pay \$300, then by the end of the year, you've paid $(12)(\$300) = \$3600$.
- This calculation is similar to the interest calculations we just did. Each month you pay $(\$100) \cdot (12\% \text{ per year})(\frac{1}{12} \text{ year}) = \$1$ interest, and at the end of the year that is $(12)(\$1) = \$12$, or 12% of the original loan.

5.1: Got no money to pay the rent

- What happens when you cannot pay the rent?
  Well, typically lots of bad things.
- What if you borrow the rent from your friend?
  Maybe he charges interest too.
- What if you can't pay back your friend?
  Maybe you borrow from another friend,
  and maybe they charge interest too.
- Interest on interest is called **compound interest**
- In finance and economics, nearly all interest is compound
- Simple interest is used for short-term loans

5.1: Three Brothers Loans and Cement Mixing, LLC

- Harry, Gary, and Scary run a limitedly legal company specializing in short term loans
- In January, Bob borrowed \$100 from Harry for one month at 10% per **month** interest
- In February, Gary stopped by to say hello, and that his brother was anxious for his \$110
- Bob didn't have the \$110, but Gary said he looked like a nice guy and would loan him the \$110 at 10% per month interest.
- Bob asked if Gary minded giving Harry the money, since they were brothers, and so Gary took back the money immediately and went back to the cement yard.

5.1: Three Brothers Loans and Cement Mixing, LLC

- In March, Scary stopped by to say hello, and that his brother was anxious for his \$121
- Bob didn't have the \$120, wait, \$121?
- If Bob had borrowed \$100 for 2 months at 10% per month interest, then he would owe:

$$\$100 + (\$100) \cdot (10\% \text{ per month}) \cdot (2 \text{ months}) = \$100 + \$20$$

However, he had borrowed \$110 from Gary for 1 month at 10% per month interest, so he owed:

$$\$110 + (\$110) \cdot (10\% \text{ per month}) \cdot (1 \text{ month}) = \$110 + \$11$$

- Scary had no interest in the math, only in the interest, \$10 from the first month, \$11 from the second

5.1: Compound interest

- The most basic formula for compound interest is:

$$A = P(1 + i)^n$$

- the **Principal** is the amount initially borrowed, like \$100
- the **interest rate** per compounding period, like 10% per month
- the **number** of compounding periods that have passed, like 2 months
- the **Accumulated Amount** of money due, both the principal and the interest, like

$$(\$100)(1 + 10\%)^2 = (\$100)(1.10)^2 = \$121$$

5.1: Compound interest examples

$$A = P(1+i)^n$$

- If you borrow \$100 at 10% per month, compounded monthly, for six months you owe

$$(\$100) \cdot (1.1)^6 \approx \$177.16$$

- If you borrow \$100 at 10% per month, compounded monthly, for nine months you owe

$$(\$100) \cdot (1.1)^9 \approx \$235.79$$

- If you borrow \$100 at 10% per month, compounded monthly, for twelve months you owe

$$(\$100) \cdot (1.1)^{12} \approx \$313.84$$

5.1: Why does the formula work?

- If you borrow \$100 at 10% per month, compounded monthly, for one month you owe

$$\$100 + (\$100)(10\%) = (\$100)\cdot(1+10\%) = (\$100)\cdot(1.1) = \$110$$

- If you borrow it for another month, you owe

$$\$110 + (\$110)(10\%) = (\$110)(1+10\%) = (\$110)(1.1)$$

$$= (\$100)(1.1)(1.1) = (\$100)(1.1)^2 = \$121$$

- If you borrow it for another month, you owe

$$\$121 + (\$121)(10\%) = (\$121)(1+10\%) = (\$121)(1.1)$$

$$= (\$100)(1.1)^2(1.1) = (\$100)(1.1)^3 = \$133.10$$

5.1: Confusing customers for fun and profit

- Stating interests rates in terms of months, fortnights, or furlongs makes it hard to compare interest rates
- A simple way to handle this is to multiply the rate by how many periods there are per year, to "convert" to a yearly rate, like $(10\% \text{ per month}) \cdot (12 \text{ months per year}) = 120\% \text{ per year}$
- The **nominal rate** is this rate, "120% interest per year, compounded monthly"
- To convert from a nominal rate to a per-period rate just divide by the number of periods
- a nominal rate of 12% per year compounded monthly is a rate of $(12\% \text{ per year})/(12 \text{ months per year}) = 1\%$ per month

5.1: Confusing customers for fun and profit

- However, what happens to Bob (best-case scenario) if he continues to get loans from the three brothers?
- The nominal rate was 120% per year, compounded monthly

Jan	Feb	Mar	Apr	May	Jun
\$100	\$110	\$121	\$133.10	\$146.41	\$161.05
Jul	Aug	Sep	Oct	Nov	Dec
\$177.16	\$194.87	\$214.36	\$235.79	\$259.37	\$285.31
Jan					
\$313.84	= \$100 +	\$213.84			

- "120% per year compounded monthly" fails to capture the "213.84% per year simple interest"

5.1: Effective interest rate

- In the U.S. the 1968 Truth in Lending Act required lenders to advertise the **effective** annual percentage rate
- The true calculation is complicated, depends on your jurisdiction, and takes into account certain fees and penalties.
- In MA162, the formula is not so complicated. You just calculate the interest for one year.
- For instance, the three brothers nominal rate of 120% resulted in

  $(1+\frac{1.20}{12})^{12}-1 = (1+0.10)^{12}-1 = 1.1^{12}-1 \approx 2.13843 = 213.843\%$

- In general

$$r_{eff} = (1 + \frac{r}{m})^m$$

5.1: Summary

- Today we learned **simple interest**, **compound interest**, and the **effective interest rate**.
- We used the words **interest**, **principal**, **interest rate**, **compounding period**, **nominal rate**, **accumulated amount**.
- You are now ready to complete HWC1 #s 1,5,6,7,8,9,10,12,13
- Make sure to take advantage of office hours, and have your questions ready for your next recitation