

MA162: Finite mathematics

Jack Schmidt

University of Kentucky

November 7, 2011

SCHEDULE:

- HW 6B is due Wednesday, Nov 9th, 2011.
- HW 6C is due Friday, Nov 11th, 2011.
- Exam 3 is Monday, Nov 14th, 5:00pm-7:00pm in CB106.

Today we will cover 6.3: Multiplication principle

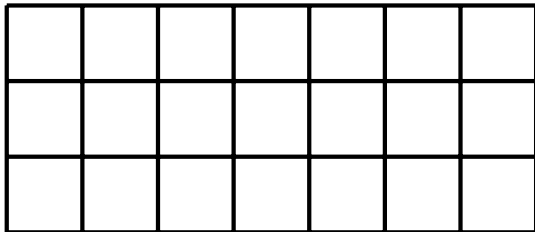
Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
 - Simple interest
 - Compound interest
 - Sinking funds
 - Amortized loans
- Chapter 6, Counting
 - Inclusion exclusion
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 - **Multiplication principle**
 - Permutations and combinations



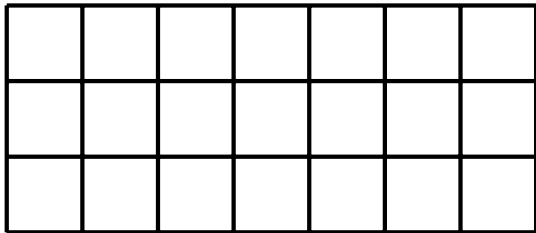
6.3: What is multiplication?

- How many squares in this figure?



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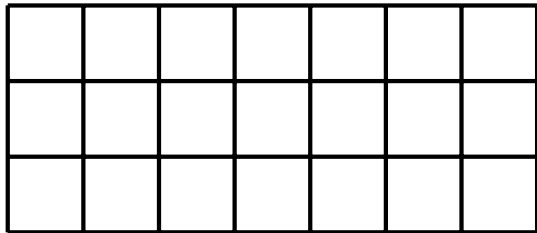
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- Each column has 3 squares, there are 7 columns, so $3 \cdot 7 = 21$

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- Each column has 3 squares, there are 7 columns, so $3 \cdot 7 = 21$
- Counting each square is slower and error-prone.

6.3: Three square meals a day

- You decide to brush your teeth after every meal, but are worried about the toothpaste consumption. You use about 1% of the tube every time you brush. How many weeks will it last?

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Brk							
Lun							
Din							

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- So 21 brushes per week; takes less than 5 weeks to use up a tube.

6.3: A rainbow of possibilities

- You are working on a dazzling fashion project and have seven dyes: **Red**, **Orange**, **Yellow**, **Green**, **Blue**, **Indigo**, and **Violet**. You've got three types of fabric: Burlap, Cotton, and Denim.

How many different color/texture combinations do you have?

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- Again $(3)(7) = 21$

	Red	Ora	Yel	Gre	Blu	Ind	Vio
Bur							
Cot							
Den							

6.3: Counting with no overlaps

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- Suppose you are doing a study on primacy and its effect on critical comparisons, so you need to convince a bunch of your film critic friends to go see a movie at each theater, but you care which theater they go to first. How many possibilities are there?

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• A picture is easier:

	1	2	3	4	5	6	
1	11	12	13	14	15	16	36 ways
2	21	22	23	24	25	26	
3	31	32	33	34	35	36	
4	41	42	43	44	45	46	
5	51	52	53	54	55	56	
6	61	62	63	64	65	66	

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- Get a **penny**, a **nickel**, and a **dime**. Flip all three.

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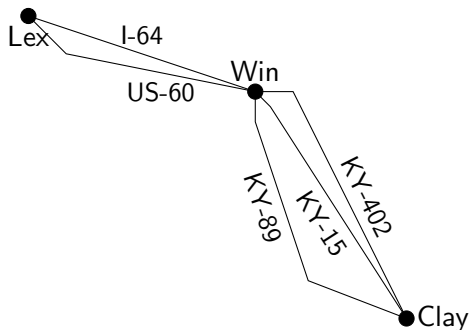
-
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How many possibilities?

- HHH**, **HHT**, **HTH**, **HTT**, **THH**, **THT**, **TTH**, **TTT**
 $(2)(2)(2)=8$

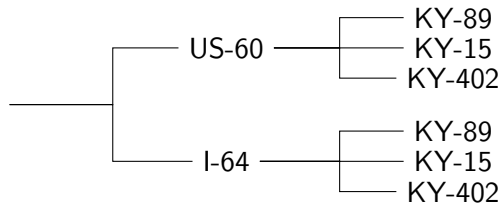
6.3: Drawing the possibilities

- There are two main ways to get to Winchester from Lexington: Winchester Rd (US-60) and I-64. From Winchester, there are three main ways to Clay City: KY-89, KY-15, and the Mountain Parkway (KY-402). How many different ways are there from Lexington to Clay City using these routes?



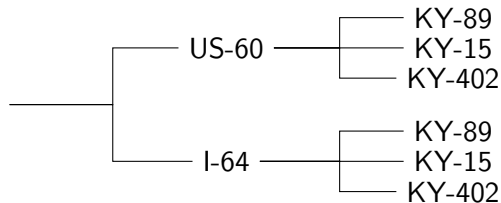
6.3: Trees for counting

- We can unfold the map to make the possibilities clearer:



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- This is a decision tree. Note how the decision to be made after I-64 is the same as the decision to be made after US-60. The first choice does not affect the second choice. The choices are **independent**.

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- $(10) \cdot (10) \cdot (10) \cdot (26) \cdot (26) \cdot (26) = 17,576,000$
- How many cars are in Kentucky?
- 4 million people, about 4 million vehicles, 2 million of which probably have standard plates

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- $(5)(10)(6) \cdot (4)(9)(5) = 54000$.

6.3: Rearranging letters

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- **RGB**, **RBG**, **GRB**, **GBR**, **BRG**, **BGR**

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- **RGB, RBG, GRB, GBR, BRG, BGR**
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and for each first letter, two choices for second (the other two),
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OH, OR, OS, OE, OY,
RH, RO, RS, RE, RY,
SH, SO, SR, SE, SY,
EH, EO, ER, ES, EY,
YH, YO, YR, YS, YE

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YH, YO, YR, YS, YE
- Six possibilities for first (H,O,R,S,E,Y)
and five for second (the remaining five)