

DEPARTMENT OF MATHEMATICS

Ma 162 Third Exam November 14, 2011

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	12	
2	12	
3	12	
4	14	
5	12	
6	14	
7	12	
8	12	
Total	100	

NAME: _____ Section: _____

Last four digits of Student ID: _____

- Simple Interest: $I = Prt$. Accumulation: $A = P(1 + rt)$.
- Compound Interest Accumulation: $A = P(1 + i)^n$. Present value: $P = A(1 + i)^{-n}$.
- Effective rate: $r_{eff} = (1 + \frac{r}{m})^m - 1$.
- Annuity: Sum: $S = R \frac{(1+i)^n - 1}{i}$. Present value: $P = R \frac{1 - (1+i)^{-n}}{i}$.
- Set counting: Two sets: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
 $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

Make sure to show formulas. Answers with no work receive no credit.

1. Mr. Marjoram is temporarily short on money, but will have plenty in a week or two. His \$80 electrical bill is due too soon, and he contemplates four options:

- (A) Pay it late, including a \$4 late fee
- (B) Put it on his 24% APR credit card for one month (incurring 2% simple interest)
- (C) Get a loan from the pawn shop for 1% monthly interest and a \$5 processing fee
- (D) Get a loan from Chek-N-Go at 432% APR for two weeks (incurring 16.80% simple interest)

How much interest does each option incur? Which is the cheapest option?

(A) Paying it late results in dollars above the original \$80

(B) Putting it on the credit card results in dollars above the \$80

(C) The loan from the pawn shop results in dollars above the \$80

(D) The loan from Chek-N-Go results in dollars above the \$80

The cheapest option is:

Make sure to show formulas. Answers with no work receive no credit.

2. Mrs. Oregano just received notification that her interest rate is changing from 12% APR to 24% APR, effective in three months. She expects to incur interest for the next six months. Assuming no further changes, how much interest will \$250.00 incur over the next six months: that is three months at 12% APR and three months at 24% APR, all compounded monthly.

(a) After three months, the accumulated amount is dollars.

(b) After six months, the accumulated amount is dollars.

(c) The interest after six months is dollars.

Mrs. Oregano has a limited time offer to transfer the present \$250.00 to an 18% APR account. How much interest would the \$250.00 incur after six months at 18% APR, compounded monthly?

(d) The accumulated amount after six months would be dollars.

(e) The interest would be dollars.

Make sure to show formulas. Answers with no work receive no credit.

3. Zach Crusoe is saving for the future. He has deposited \$0.10 per day into his 3.60% APR savings account (compounded daily, 360 days per year) for two years. How much is his account currently worth?

(a) His account is worth dollars.

As he has gotten older, his responsibilities and allowance have increased. How much will his account be worth if he now deposits \$0.25 per day for the next year?

(b) His account is worth dollars after 3 years: 2 years of \$0.10 per day, and 1 year of \$0.25 per day

Make sure to show formulas. Answers with no work receive no credit.

4. Dr. Tarragon is buying his potatoes on credit and plans to purchase \$1000.00 worth of Yukon Golds at 18% APR compounded monthly. He needs to have them paid off by the end of the year, 9 months from now. How much is his monthly payment?

(a) A monthly payment of dollars will pay off the loan in 9 months.

(b) Dr. Tarragon realizes he can only afford half that much. Rather than borrow half as much, he decides to pay it back half as quickly. A monthly payment of dollars will pay off the loan in 18 months.

(c) Actually, that is still too expensive. How long would it take to pay it off at \$15 per month? After one month, the debt is dollars, so after the payment the remaining debt is dollars.

(d) What is the remaining debt after 6 years? (This is when he has paid approximately the same amount in total payments) After six years of \$15 monthly payments, he still owes dollars.

Make sure to show formulas. Answers with no work receive no credit.

5. Suppose that L , M and N are sets with 41, 38, and 40 members respectively. Calculate the indicated quantities. Display correct formulas or appropriate Venn diagrams.

(a) If $L \cup M$ has 62 members, then $L \cap M$ has members.

(b) If it is further known that $L \cap N$ has 19 members, then $L \cup N$ has members.

(c) If, in addition, $M - N$ has 20 members, then $M \cap N$ has members.

(d) Finally, if we are given that the intersection of all three sets L , M , and N has 10 members, then the union of these three sets has members.

Make sure to show formulas. Answers with no work receive no credit.

6. A survey of 100 College students were asked for their opinions about pizza. They were specifically whether they liked pepperoni, mushrooms, and garlic.

- 42 students liked pepperoni.
- 41 students liked mushrooms.
- 36 students liked garlic.
- 13 students liked both pepperoni and mushrooms.
- 11 students liked both pepperoni and garlic.
- 9 students liked both mushrooms and garlic.
- 6 students liked all three toppings.

Based on the above information, answer the following questions. You must show your calculations to receive credit.

(a) How many students surveyed did not like any of the three toppings?

Answer:

(b) How many students surveyed liked at least two of the toppings?

Answer:

Make sure to show formulas. Answers with no work receive no credit.

7. Your neighbor's daughter is putting on a fashion show for the neighborhood. She owns 4 pairs of pants, 10 shirts, and 3 hats, and plans to model every combination for her audience.

How many outfits does she plan to model in the show?

Answer: outfits

Before the show begins, you find an opportunity to shorten the show by hiding one article of clothing (lowering the number of combinations of outfits) but you only have time to hide one. To remove the highest number of possible outfits, should you hide a shirt, a pair of pants, or a hat?

Answer:

How many outfits would be modeled now? **Answer:** outfits

Make sure to show formulas. Answers with no work receive no credit.

8. A fashion-conscious spider has 8 legs (4 pairs of feet) and 12 pairs of shoes. Assuming the spider always wears shoes in pairs, how many ways can she get dressed in the morning?

(a) **Answer:** outfits

What if the spider is exploring more avant-garde fashion styles and decides the shoes do not need to match. Of course, she only wears left shoes on the left feet, and right shoes on the right feet, but doesn't care if the left shoe matches the right shoe. How many outfits are now available?

(b) **Answer:** outfits

What if the spider has decided matching shoes are for insects, and refuses to have any matching shoes, even on different feet! Of course left shoes are on the left feet, and right shoes are on the right feet. How many outfits are now available?

(c) **Answer:** outfits

Answers:

1a. The late fee is the interest. \$4.

1b. $\$80(0.02) = \1.60 is the interest. One month of compound interest compounded monthly is the same as one month of simple interest.

1c. $\$80(0.01) + 5 = \5.80 is the interest, including the fee.

1d. $\$80(0.1680) = \13.44 is the interest. This interest rate is taken from a similar company's website.

1e. The cheapest option is the credit card.

2a. This is just compound interest. $P = \$250.00$, $r = 0.12/12 = 0.01$, $n = 3$,

$$A = \$250.00(1.01)^3 = \$257.58$$

2b. This is also just compound interest: $P = \$257.58$, $r = 0.24/12 = 0.02$, $n = 3$,

$$A = \$257.58(1.02)^3 = \$273.34$$

One can also just calculate this in one go:

$$A = \$250.00(1.01)^3(1.02)^3 = \$273.34$$

2c. The interest is just the extra money: $I = \$273.34 - \$250.00 = \$23.34$

2d. This is just compound interest. Notice how the answer is almost the same, but the higher interest rate earlier is more important than the lower interest rate later. Generally speaking, the earlier part of a longterm loan is the most influential.

$$A = \$250.00(1.015)^6 = \$273.36$$

2e. $I = \$273.36 - \$250.00 = \$23.36$

3a. We need money later, and are making recurring payments, so this is the accumulated value of annuity. $R = \$0.10$, $n = 720$, $r = 0.0360/360 = 0.0001$,

$$A = R((1 + r)^n - 1)/r = \$0.10(1.0001^{720} - 1)/0.0001 = \$74.65$$

3b. This is tricky because the recurring payment changes. One way to handle this is as two annuities: one runs for three years ($n = 1080$) and is \$0.10 per day. One runs only for one year ($n = 360$) and is \$0.15 per day (the rest of the \$0.25). We get:

$$A = \$0.10(1.0001^{1080} - 1)/0.0001 + \$0.15(1.0001^{360} - 1)/0.0001 = \$169.02$$

One could also do this as an annuity, a compound interest, and an annuity. we take the \$74.65 from part a, and let it sit in the bank for one year: $\$74.65(1.0001)^{360} = \77.39 . Then we add to it the \$0.25 per day annuity for one year:

$$A = \$77.39 + \$0.25(1.0001^{360} - 1)/0.0001 = \$169.02$$

4a. We need money now, and are making recurring payments, so this is the present value of an annuity. $P = \$1000$, $n = 9$ months, $r = 0.18/12 = 0.015$, $R = ?$

$$\$1000 = R(1 - 1.015^{-9})/(0.015) \quad R = \$1000/((1 - 1.015^{-9})/(0.015)) = \$119.61$$

4b. Just change $n = 18$. However, the payment is more than just half of the old one (which would be \$59.81).

$$\$1000 = R(1 - 1.015^{-18})/(0.015) \quad R = \$1000/((1 - 1.015^{-18})/(0.015)) = \$63.81$$

4c. This is just simple interest (or compound interest with $n = 1$). $P = \$1000$, $r = 0.015$, $n = 1$, $A = \$1000(1.015) = \1015 , and after the payment of \$15, exactly \$1000 is left. The payment has just brought the debt back to the original amount. All he has done is pay the interest.

4d. After six years he has paid a total of \$1080 (while he would have only paid a total of \$1076.49 over the nine months in part a), and yet his debt is still exactly \$1000. It will never be paid off at this rate.

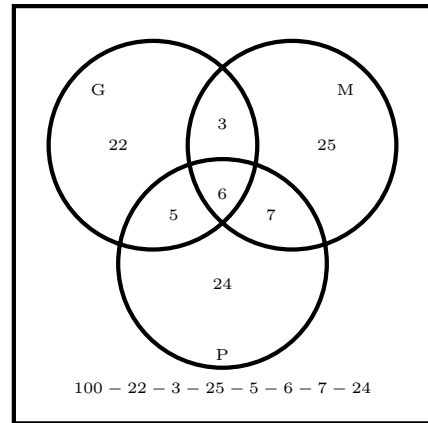
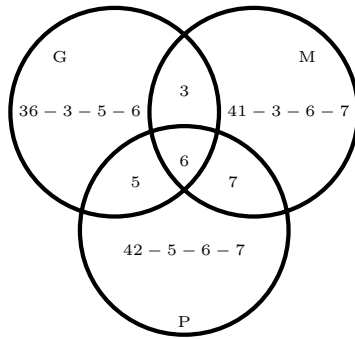
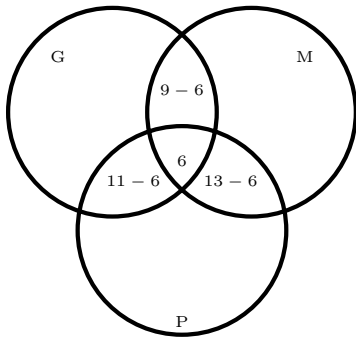
5a. This is done with the formulas on the front. $62 = 41 + 38$ is not quite right. When we add the 41 and the 38 we must be counting some people twice: $41 + 38 - 62 = 17$ people are in both L and M , and so are counted twice. This is just $n(L) + n(M) - n(L \cup M) = n(L \cap M)$.

5b. This is similar. We add 41 and 40, but that counts 19 people twice, so we only end up with $41 + 40 - 19 = 62$ total.

5c. This formula is not given, but is easy: the people in M are either in N (so in $M \cap N$) or not (so in $M - N$). That means the 38 people are divided into a group of 20 and a group of $38 - 20 = 18$ people.

5d. Here we just use the inclusion-exclusion formula from the front: $41 + 38 + 40$ counts some people twice. $41 + 38 + 40 - 17 - 19 - 18$ removes one copy of the double counted people, but it removes three copies of the triple counted people, so we still need to count them once: $41 + 38 + 40 - 17 - 19 - 18 + 10 = 75$ total when everyone is counted exactly once.

6. This can be done using a Venn diagram. 9 like mushrooms and garlic, 6 like all three, so apparently $9 - 6 = 3$ like mushrooms and garlic but not pepperoni. Handling all pairs of toppings gives the left diagram. Since 36 people like mushrooms, but 3, 6, and 5 of them like other toppings too, that leaves $36 - 3 - 6 - 5$ people who only like mushrooms. Handling all single toppings this way gives the middle diagram. Finally, we count people who like any particular topping to find those that like none, giving the last diagram. Thus $100 - 22 - 3 - 25 - 5 - 6 - 7 - 24 = 8$ people did not like any of the toppings, and $3 + 5 + 6 + 7 = 21$ liked at least two toppings.



7a. Her sense of fashion is not yet developed, and so all hats match all shirts and all pants, giving $(3)(4)(10) = 120$ outfits.

7b. Removing hats eliminates 40 outfits per hat, much better than 30 per shirt, or 12 per pair of pants. This leaves $(3 - 1)(4)(10) = 80$ outfits.

8a. Each pair of feet gets matched to a pair of shoes: $(12)(11)(10)(9) = 11880$ outfits.

8b. The left feet and right feet get dressed independently. 11880 outfits for the left, times 11880 outfits for the right: $(11880)(11880) = 141134400$.

8c. Now there are eight feet, and twelve pairs of shoes (where we only take one shoe from each pair, either the left or right as appropriate) giving: $(12)(11)(10)(9)(8)(7)(6)(5) = 19958400$ outfits.