

MA162: Finite mathematics

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University of Kentucky

November 8, 2011

SCHEDULE:

- HW 6B is due Wednesday, Nov 9th, 2011.
- HW 6C is due Friday, Nov 11th, 2011.
- Bring practice exam on Monday.
- Exam 3 is Monday, Nov 14th, 5:00pm-7:00pm in CB106.

Today we will cover 6.4: Permutations

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
 - Simple interest
 - Compound interest
 - Sinking funds
 - Amortized loans
- Chapter 6, Counting
 - Inclusion exclusion
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 - Multiplication principle
 - **Permutations**



6.4: Trifecta!

- Some people bet on horse races, a “Trifecta” bet is common
- You predict the first, second, and third place winners, in order.
- There are 14 contenders: **A**ccounting We Will Go, **B**usiness Planner, **C**orporate Finance, **D**ebt Sealing, **E**conomy Model, **F**iscal Filly, **G**ross Domestic Pony, **H**orse Resources, **I**nitial Pony Offering, **J**ust Another Horsey, **K**arpay Deeum, **L**OL Street, **M**arkety Mark, and **N**o Chance Vance
- Which ones will you choose? **A, B, C** or **L, N, E**?
- How many possibilities?

6.4: Counting the possibilities

_____ _____ _____
1ST 2ND 3RD

- There are three places

6.4: Counting the possibilities

$$\begin{array}{ccc} 14 & & \\ \hline 1^{\text{ST}} & 2^{\text{ND}} & 3^{\text{RD}} \end{array}$$

- There are three places
- There are 14 possibilities for first place,

6.4: Counting the possibilities

$$\begin{array}{ccc} 14 & 13 & \\ \hline 1^{\text{ST}} & 2^{\text{ND}} & 3^{\text{RD}} \end{array}$$

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place

6.4: Counting the possibilities

$$\begin{array}{ccc} 14 & 13 & 12 \\ \hline 1^{\text{ST}} & 2^{\text{ND}} & 3^{\text{RD}} \end{array}$$

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place
- and only 12 left for third place

6.4: Counting the possibilities

$$\frac{14}{1^{\text{ST}}} \quad \frac{13}{2^{\text{ND}}} \quad \frac{12}{3^{\text{RD}}} = 2184$$

- There are three places
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- and only 12 left for third place
- That is $(14)(13)(12) = 2184$ total possibilities

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- and only 12 left for third place
- That is $(14)(13)(12) = 2184$ total possibilities
- If you bet 1000 times, only a 1 in 3 chance of winning at least once

6.4: Club officers

- The Variety Club has a President, a Vice President, a Secretary, and a Treasurer
- The V.C. has 6 members: Art, Ben, Cin, Dan, Eve, and Fin.
- But every day they want to assign a different set of officers
- Can they make it a year without exactly repeating the officer assignments?
- So maybe ABCD, then ABCE, then ABCF, then ABDC, then ...

6.4: Counting the assignments

Pres Vice Sec. Trs.

- There are four positions, and order matters

6.4: Counting the assignments

$$\frac{6}{\begin{array}{cccc} \hline & & & \\ \hline \textit{Pres} & \textit{Vice} & \textit{Sec.} & \textit{Trs.} \\ \hline \end{array}}$$

- There are four positions, and order matters
- There are 6 people available to president each day

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$$\frac{6}{\text{Pres}} \quad \frac{5}{\text{Vice}} \quad \frac{\quad}{\text{Sec.}} \quad \frac{\quad}{\text{Trs.}}$$

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP

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$$\begin{array}{cccc} 6 & 5 & 4 & \\ \hline \textit{Pres} & \textit{Vice} & \textit{Sec.} & \textit{Trs.} \end{array}$$

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary

6.4: Counting the assignments

$$\begin{array}{cccc} 6 & 5 & 4 & 3 \\ \hline \textit{Pres} & \textit{Vice} & \textit{Sec.} & \textit{Trs.} \end{array}$$

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer

6.4: Counting the assignments

$$\frac{6}{\text{Pres}} \quad \frac{5}{\text{Vice}} \quad \frac{4}{\text{Sec.}} \quad \frac{3}{\text{Trs.}} = 360$$

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer
- There are $(6)(5)(4)(3) = 360$ possible assignments

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- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer
- There are $(6)(5)(4)(3) = 360$ possible assignments
- Not enough for a calendar year, but certainly for a school year!

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$$\begin{array}{r} 10 \\ \hline \textit{Pres} \end{array} \quad \begin{array}{r} \\ \hline \textit{Trs.} \end{array}$$

- There are ten people eligible for president

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- But only five people left for vice president

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$$\frac{10}{\text{Pres}} \quad \frac{5}{\text{Trs.}} = 50$$

- There are ten people eligible for president
- But only five people left for vice president
- That is $(5)(10) = 50$ different officer assignments

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- Well, there are ten shoes trying out for the first part, but whomever you choose also eliminates their stunt double
- So eight for the second part, and six for the third; $10 \cdot 8 \cdot 6 = 480$ ways.

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- But you don't care what order they are in. So that is four ways:

$$\{L1, L2\}, \{L1, R2\}, \{R1, L2\}, \{R1, R2\}$$

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- 4×2 ways counting order, then divide by two to ignore order

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- How many ways can one rearrange the letters of KENTUCKY?
- Well, a little different since there are two Ks
- $8!$ ways if we keep track of which K is which, then divide by two since each word like KENTUCKY appears twice as kENTUCKY and KENTUCKy.

$$8!/2 = 20160$$

6.4: Team players

- If there are 15 able bodied players, and we need to choose 11 of them to be on the field. We want four forwards, three midfielders, three defenders, and one goalie. We let the players themselves dynamically decide on the left/right/center. How many selections are possible?

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- If there are 15 able bodied players, and we need to choose 11 of them to be on the field. We want four forwards, three midfielders, three defenders, and one goalie. We let the players themselves dynamically decide on the left/right/center. How many selections are possible?
- $(15)(14)(13)(12)$ choices of forwards counting order, but $(4)(3)(2)(1)$ ways of re-ordering them, so $(15)(14)(13)(12)/((4)(3)(2)(1)) = 15!/(11!4!) = 1365$ ways ignoring order

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- Then 5 ways of choosing the goalie.

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- Then $8!/(5!3!) = 56$ ways of choosing defenders ignoring order
- Then 5 ways of choosing the goalie.
- Total is: $(1365)(165)(56)(5)$ ways of choosing the first string

6.4: Summary

- We learned to handle symmetries in our counting, especially **permutations**, and **combinations**.
- Make sure to complete the homework ASAP, and begin work on the practice exam
- Be ready to discuss the practice exam next class; bring a copy