MA162: Finite mathematics

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November 8, 2011

Schedule:

- HW 6B is due Wednesday, Nov 9th, 2011.
- HW 6C is due Friday, Nov 11th, 2011.
- Bring practice exam on Monday.
- Exam 3 is Monday, Nov 14th, 5:00pm-7:00pm in CB106.

Today we will cover 6.4: Permutations

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
 - Simple interest
 - Compound interest
 - Sinking funds
 - Amortized loans
- Chapter 6, Counting
 - Inclusion exclusion
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 - Multiplication principle
 - Permutations

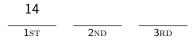




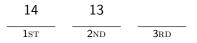
- Some people bet on horse races, a "Trifecta" bet is common
- You predict the first, second, and third place winners, in order.
- There are 14 contenders: Accounting We Will Go, Business Planner, Corporate Finance, Debt Sealing, Economy Model, Fiscal Filly, Gross Domestic Pony, Horse Resources, Initial Pony Offering, Just Another Horsey, Karpay Deeum, LOL Street, Markety Mark, and No Chance Vance
- Which ones will you choose? A, B, C or L, N, E?
- How many possibilities?

1st 2nd 3rd

• There are three places



- There are three places
- There are 14 possibilities for first place,



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- but only 13 left for second place

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- and only 12 left for third place

14	13	12	= 2184
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- If you bet 1000 times, only a 1 in 3 chance of winning at least once

- The Variety Club has a President, a Vice President, a Secretary, and a Treasurer
- The V.C. has 6 members: Art, Ben, Cin, Dan, Eve, and Fin.
- But every day they want to assign a different set of officers
- Can they make it a year without exactly repeating the officer assignments?
- So maybe ABCD, then ABCE, then ABCF, then ABDC, then

Pres Vice Sec. Trs.

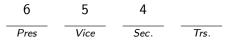
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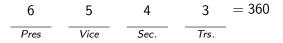
- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP



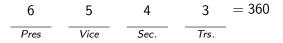
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- There are 4 people left to be Secretary

6	5	4	3
Pres	Vice	Sec.	Trs.

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- Not enough for a calendar year, but certainly for a school year!

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 $\frac{10}{Pres} \quad \frac{5}{Trs.} = 50$

- There are ten people eligible for president
- But only five people left for vice president
- That is (5)(10) = 50 different officer assignments

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- So eight for the second part, and six for the third; 10*8*6 = 480 ways.

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- But you don't care what order they are in. So that is four ways:

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• 4*2 ways counting order, then divide by two to ignore order

6.4: Spelling

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- Well, a little different since there are two Ks
- 8! ways if we keep track of which K is which, then divide by two since each word like KENTUCKY appears twice as kENTUCKY and KENTUCkY.

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- Then 5 ways of choosing the goalie.
- Total is: (1365)(165)(56)(5) ways of choosing the first string

• We learned to handle symmetries in our counting, especially **permutations**, and **combinations**.

• Make sure to complete the homework ASAP, and begin work on the practice exam

• Be ready to discuss the practice exam next class; bring a copy