MA162: Finite mathematics

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November 21, 2011

SCHEDULE:

- HW 0.0 through 7A is due Sunday, Nov 27th, 2011.
- HW 7B is due Friday, Dec 2, 2011.
- HW 7C is due Friday, Dec 9, 2011.
- Final Exam is Wednesday, Dec 14th, 8:30pm-10:30pm.

Today we will cover 7.2: Probability

Final Exam

- Chapter 7: Probability

  - Counting based probability

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  - Empirical probability

  - Conditional probability

- Cumulative

  - Ch 2: Setting up and reading the answer from a linear system

  - Ch 3: Graphically solving a 2 variable LPP

  - Ch 4: Setting up a multi-var LPP

  - Ch 4: Reading and interpreting answer form a multi-var LPP

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- There are 21 such pairs, and if all pairs are equally likely (the dice are fair), then that is $\frac{21}{36} = \frac{7}{12} \approx 58\%$

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- Explicitly:
  HHHHH, HHHHT, HHHTH, HHHTT, HHTTT, HTHHH,
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- It should be the same for getting an odd number of tails, right? Tails, heads, what is the difference?

- But you either get an odd number of heads, or an odd number of tails, and not both, so each should be about equally likely: 50%

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- If each bulb is independent, that is $(0.1\%)^{700} \approx 0\%$ chance of this happening

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- Total is: $0.844 = 84.4\%$ chance that at most one breaks, so not too bad. Every 6 weeks you'll have a light out and no replacement, but not too bad.

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- The odds of none going out is $(99.9\%)^{7000} \approx 0.1\%$,
  exactly one are $7000 \cdot (0.1\%)(99.9\%)^{6999} \approx 0.6\%$,
  exactly two are $\frac{7000 \cdot 6999}{2} \cdot (0.1\%)^2(99.9\%)^{6998} \approx 2.2\%$,
  ...

0	1	2	3	4	5	6	7	8	9	10
0.1	0.6	2.2	5.2	9.1	12.7	14.9	14.9	13.0	10.1	7.0

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- Total is: $0.902 = 90.2\%$ chance that at most ten break, so really we're even more certain to be ok now; every 10 weeks we'll be short a bulb.

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- The risk to 10,000 people is quite small, much less than 10,000 times the risk of one

Round table

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- Sample space is:
  ABCD, ABCE, ABDC, ABDE, ABEC, ABED, ACBD, ACBE, ACDB, ACDE, ACEB, ACED, ADBC, ADBE, ADCB, ADCE, ADEB, ADEC, AEBC, AEBD, AECB, AECD, AEDB, AEDC, BCDE, BCED, BDCE, BDEC, BECD, BEDC

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- The event is all those with DE or ED (be careful of wraparound)

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- 12 bad out of 30 total is 40% chance for showers (of fists)