MA162: Finite mathematics

Jack Schmidt

University of Kentucky

April 25, 2011

SCHEDULE:

- HW 7C is due Friday, Dec 9, 2011.
- Final Exam is Wednesday, Dec 14th, 8:30pm-10:30pm.

Today we will cover 7.5: Rules of probability

Final Exam Breakdown

- Chapter 7: Probability
  - Counting based probability
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  - Empirical probability
  - Conditional probability
- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP

7.5: The Punnet square of probability

- Suppose we have the following table of young men and women with and without driver's licenses:

	Yes	No	Total
M	491	9	500
F	486	14	500
T	977	23	1000

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- What are the odds a randomly selected person is female?

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- What are the odds that a randomly selected non-driver is female?

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- Are females less likely to be drivers?

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- Are females less likely to be drivers?
- Probability a female is a driver: $\frac{486}{500} = 97\%$ nearly the same

7.5: Conditional probability

- Let's redo this using the language of events:
  - M is the event the chosen person is male
  - F is the event the chosen person is female
  - Y is the event the chosen person has a driver's license
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- We calculated it as $Pr(N \cap F)/Pr(N)$
- We need a name for this calculation, **conditional probability** $Pr(F|N) = Pr(N \cap F)/Pr(N)$ is the probability of F **given** N

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- We want to compare the probabilities of $Pr(A)$ versus $Pr(A|B)$ if they are equal then the events are **independent**

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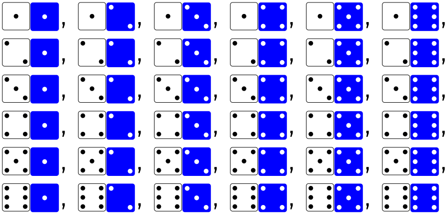

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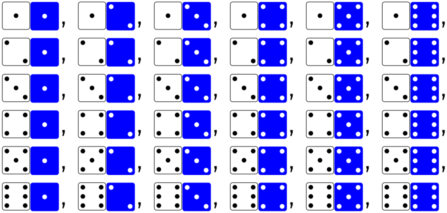

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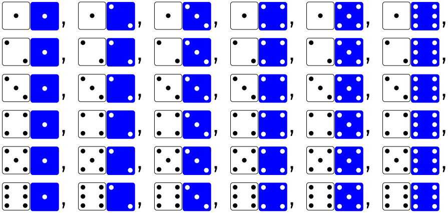

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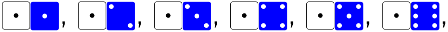

$4/6 \approx 67\%$

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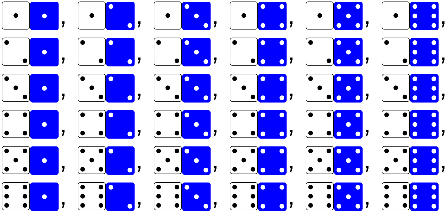

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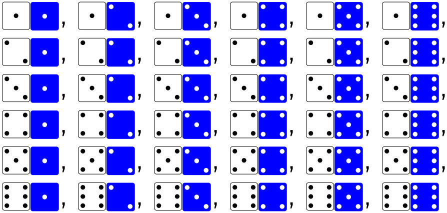

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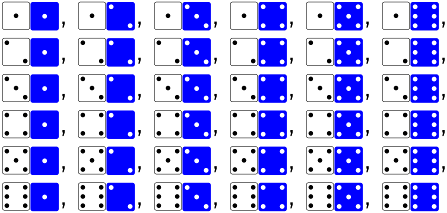

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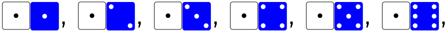

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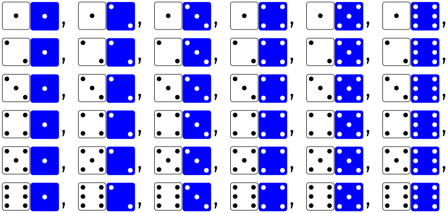

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- The first die had no effect on the outcome! The two events are said to be **independent**.

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  "Mostly". The probabilities are not equal, but they are close.

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- Weighted averages

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- 45%, right?

Reasoning backwards

- Shifty Teddy is spending some time on the gameshow "Who's Gow?" and so you have to use his pal, Shifty Eddy, to run cokes for you. You end up with a coke 30% of the time. How often does he take the money and run?

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- $Pr(F) = 30\%$, and we want to find $Pr(E)$ which we calculated to be 60%, but where do we use the 50%?

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- **Bayes's Law:** $Pr(E \cap F) = Pr(F|E) \cdot Pr(E)$ – a weighted average!

Practice exam

- A drug test is 98% accurate: out of 100 drug users, 98 will get a positive result, and 2 a negative; out of 100 non-users 98 will get a negative result, and 2 a positive. A company (somehow) knows that exactly 1 of its 100 employees is a drug user, but (somehow) does not know which one.
- An employee is picked at random to be tested, and tests positive. What is the probability that they are the drug user, given that they tested positive? Hint: It is NOT 98%.
- The company wants to be sure, and so tested the employee again. Positive. again. What is the probability that an employee is the drug user, given that they tested positive twice?

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- What is the probability that the drug test would correctly report on all 100 employees?
- An employee is picked at random to be tested twice, and tests positive once and negative once. What is the probability an employee is the drug user, given that they tested positive once and negative once?