MA162: Finite mathematics

Jack Schmidt

University of Kentucky

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Schedule:

- HW 7C is due Wednesday, Dec 14, 2011.
- Final Exam is Wednesday, Dec 14th, 8:30pm-10:30pm.

Today we will review.

Practice Exam #1

- **Setup the following system of linear equations.**
  Mr. Marjoram runs a stuffed animal factory, and is very worried about paying taxes on his rather large inventory of plush fabric, cloud-like stuffing, and whimsical trim. He decides he is going to use every last bit of his inventory to make the 2011 Marjoram Menagerie! His menagerie only includes Pandas, Saint Bernards, and Onery Ostriches. The supply requirements and availability are in the table below. How many stuffed animals of each type should he make in order to use up all of his inventory?

	Plush	Stuffing	Trim
Panda	1.5	30	12
Bernard	2	35	8
Ostrich	2.5	25	5
Inventory	110	1400	350

Practice Exam #2: Final matrix (kind of long question)

- **Interpret the final matrix for this solution to a word problem.** Vincent is trying to optimize his profit by solving a system of linear equations. He sets X to be the number of Sunshine paintings to produce, Y to be the number of Lollipop paintings to produce, A to be the tubes of Amarillo paint left over, B to be the tubes of Berry Red paint left over, C to be the number of canvasses left over, D to be tubes of Dark Blue paint left over, and P to be the profit. His decision is governed by the equations:

$$\begin{aligned} 3X+\ \ Y+A&=25\\ 3X+\ 2Y+B&=26\\ X+\ \ Y+C&=10\\ X+\ 3Y+D&=24\\ 10X+12Y\ \ &=P \end{aligned}$$

Practice Exam #2 continued

- Converting this to a matrix, he quickly reduced this to something very similar to RREF:

$$\left(\begin{array}{cc|cccc|c|c} X & Y & A & B & C & D & P & rhs \\ \hline 3 & 1 & 1 & 0 & 0 & 0 & 0 & 25 \\ 3 & 2 & 0 & 1 & 0 & 0 & 0 & 26 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 10 \\ 1 & 3 & 0 & 0 & 0 & 1 & 0 & 24 \\ \hline -10 & -12 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow[\substack{R_4(1/3)R_1 \\ R_5+(10/3)R_1}]{\substack{R_1/(3) \\ R_2R_1 \\ R_3(1/3)R_1}} \left(\begin{array}{cc|cccc|c|c} X & Y & A & B & C & D & P & rhs \\ \hline 1 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 25/3 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 2/3 & -1/3 & 0 & 1 & 0 & 0 & 5/3 \\ 0 & 8/3 & -1/3 & 0 & 0 & 1 & 0 & 47/3 \\ \hline 0 & -26/3 & 10/3 & 0 & 0 & 0 & 1 & 250/3 \end{array}\right) \xrightarrow[\substack{R_4(8/3)R_2 \\ R_5+(26/3)R_2}]{\substack{R_1(1/3)R_2 \\ R_3(2/3)R_2}}$$

$$\left(\begin{array}{cc|cccc|c|c} X & Y & A & B & C & D & P & rhs \\ \hline 1 & 0 & 2/3 & -1/3 & 0 & 0 & 0 & 8 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1/3 & -2/3 & 1 & 0 & 0 & 1 \\ 0 & 0 & 7/3 & -8/3 & 0 & 1 & 0 & 13 \\ \hline 0 & 0 & -16/3 & 26/3 & 0 & 0 & 1 & 92 \end{array}\right) \xrightarrow[\substack{R_4(7)R_3 \\ R_5+(16)R_3}]{\substack{R_1(2)R_3 \\ R_2+(3)R_3 \\ (3)R_3}} \left(\begin{array}{cc|cccc|c|c} X & Y & A & B & C & D & P & rhs \\ \hline 1 & 0 & 0 & 1 & -2 & 0 & 0 & 6 \\ 0 & 1 & 0 & -1 & 3 & 0 & 0 & 4 \\ 0 & 0 & 1 & -2 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & -7 & 1 & 0 & 6 \\ \hline 0 & 0 & 0 & -2 & 16 & 0 & 1 & 108 \end{array}\right) \xrightarrow[\substack{R_4/(2) \\ R_5+R_4}]{\substack{R_1(1/2)R_4 \\ R_2+(1/2)R_4 \\ R_3+R_4}}$$

Practice Exam #2 continued

$$\left(\begin{array}{cc|cc|rr|c|r} X & Y & A & B & C & D & P & rhs \\ \hline 1 & 0 & 0 & 0 & 3/2 & -1/2 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1/2 & 1/2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -4 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & -7/2 & 1/2 & 0 & 3 \\ \hline 0 & 0 & 0 & 0 & 9 & 1 & 1 & 114 \end{array}\right)$$

- Which variables are free?
- Convert the last row to an equation, and solve it for the non-free variable.
- What value should the free variables have to maximize P? (assuming they cannot be negative)
- Solve the third row for a non-free variable, and replace the free variables by their values from part (c).

Practice Exam #3: Practical

- During Winter Vacation your pal Vincent decides to start his own roadside art business to fund a action-packed road trip to the Bahamas. He may have drifted off in art class most days, but he did learn to draw a pretty awesome Sunshine! and some sweet Lollipops. The requirements and profits of his two painting styles are given in the following table:

	Amarillo	Berry Red	Canvasses	Dark Blue	Profit
Sunshine!	3	3	1	1	10
Lollipops	1	2	1	3	12
Inventory	25	26	10	24	

- How many Sunshine! paintings and Lollipop paintings should Vincent produce in order to maximize his profit? How much of his supplies are left over?

Practice exam #4: Graphical method

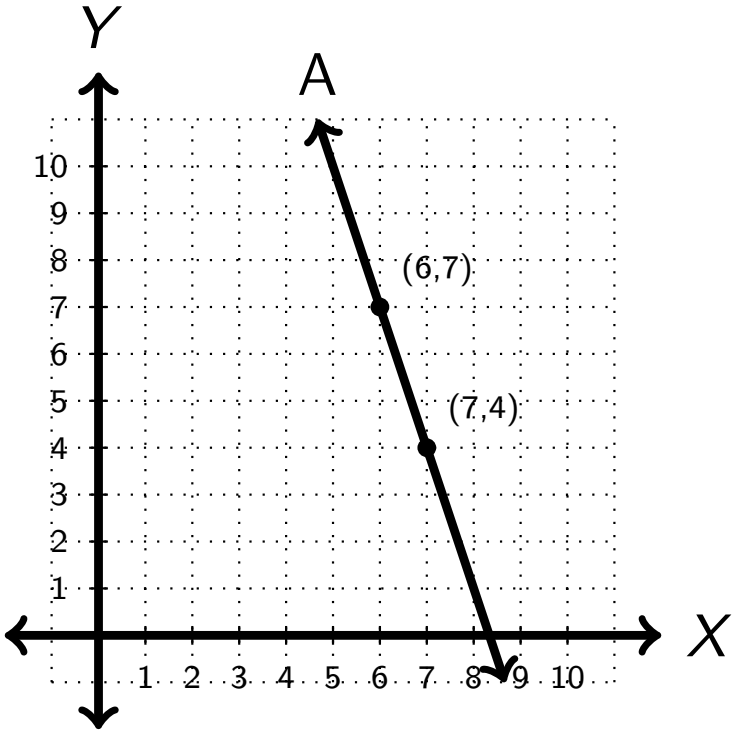
- **Completely solve the following LPP using the graphical method.** Graph the feasible region for the following LPP. You will be graded on three aspects: correctly drawn edges, correctly shaded region, and correctly labelled corners. List the corners, determine if the region is bounded or unbounded, and find the maximum value of P.

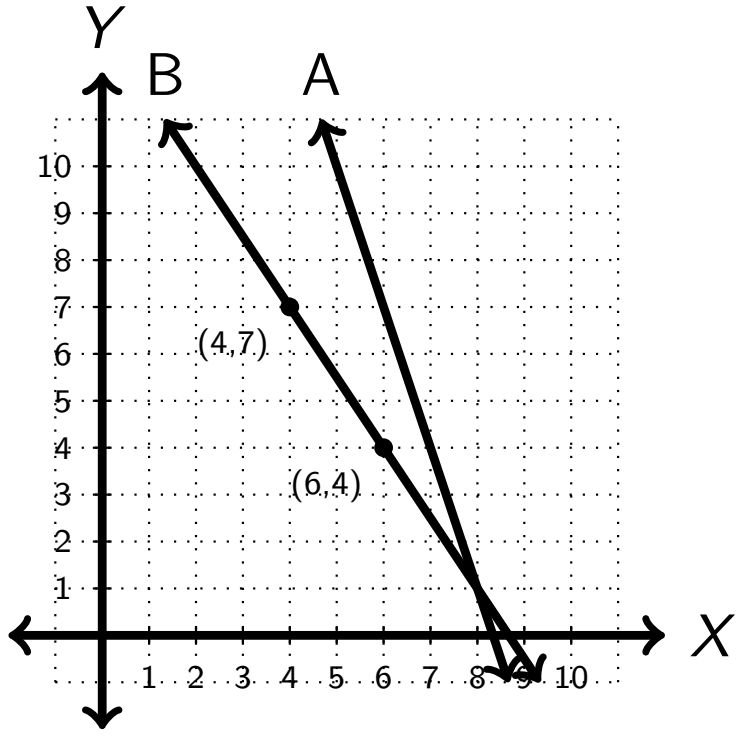
- Maximize $P = 10x + 12y$ subject to $\left\{ \begin{array}{r} 3x+\ y \leq 25 \\ 3x+2y \leq 26 \\ x+\ y \leq 10 \\ x+3y \leq 24 \end{array} \right\}$ and

  $x \geq 0$, $y \geq 0$.

Practice exam #4: Solution part 1

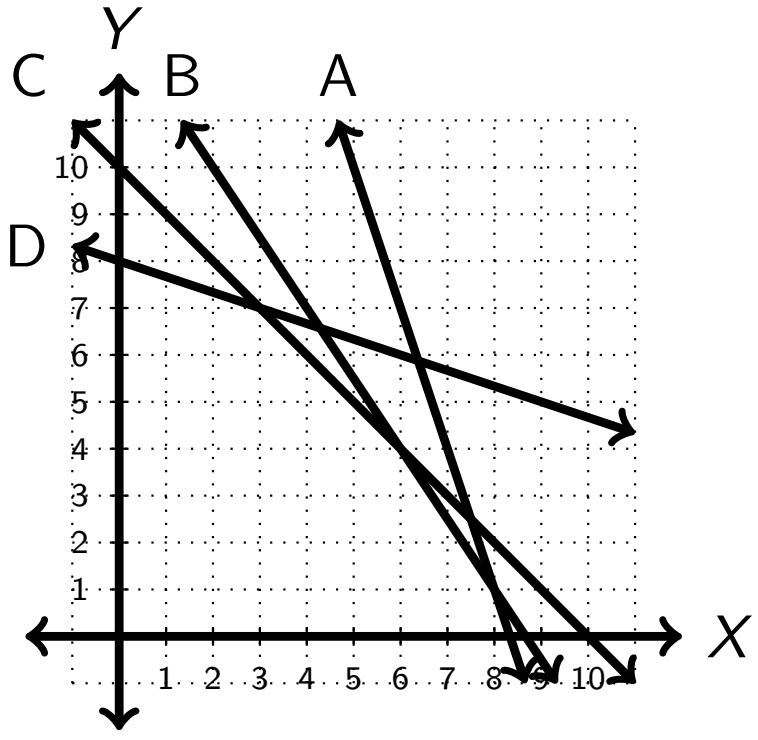
- Draw $A : 3x + y = 25$ by plugging in $x = 6$ to get $y = 7$, and plugging in $x = 7$ to get $y = 4$. That is two points $(6, 7)$ and $(7, 4)$, and there is only one line that goes through both.
- Draw $B : 3x + 2y = 26$ by plugging in $x = 6$ to get $y = 4$, and then plugging in $x = 4$ to get $y = 7$; that is two points $(6, 4)$ and $(4, 7)$ and B is the line between them.





Practice exam #4: solution 2

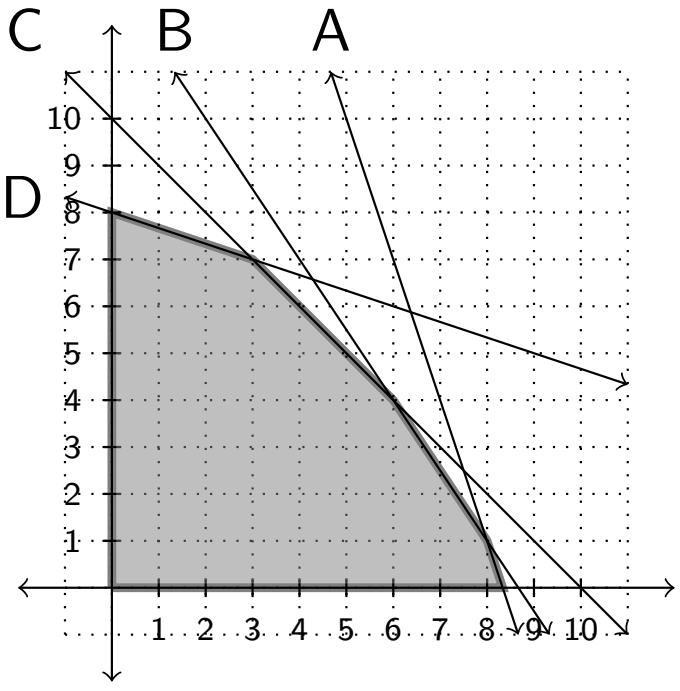
- With all lines drawn:

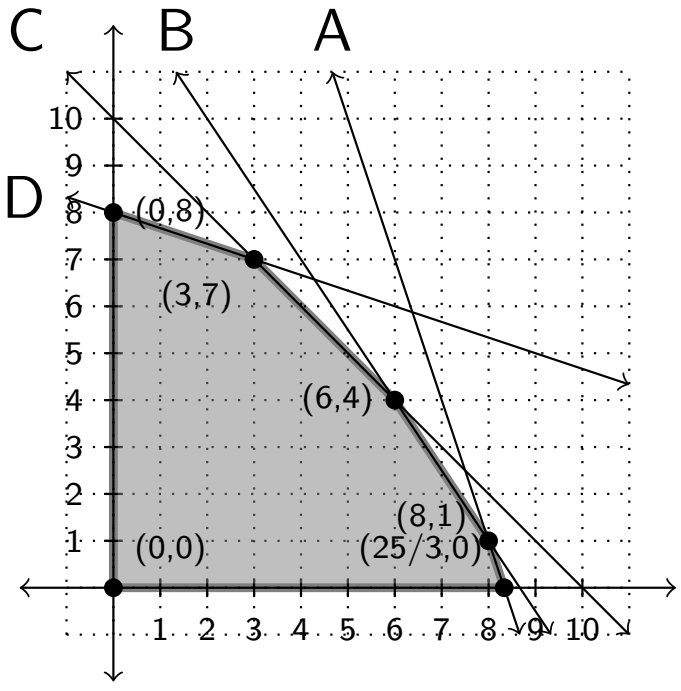


- Now test which region is correct. Plugging $(x = 1, y = 1)$ into all of the inequalities always results in true statements. For example $A : 3(1) + (1) \leq 25$ since $3 \leq 25$, etc. Hence the correct region to shade is the region with $(1, 1)$ inside.

Practice exam #4: solution part 3

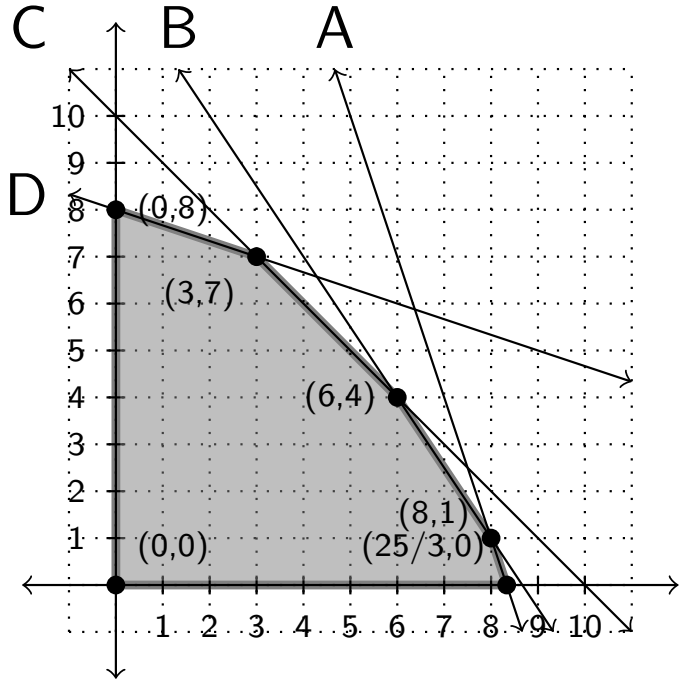
- Now find the corners by intersecting lines. The intersection of $D : x + 3y = 24$ with $C : x + y = 10$; subtracting the two equations gives $2y = 14$, so $y = 7$ and $x + 7 = 10$, so the intersection is $(x = 4, y = 7)$.





Practice exam #4: solution 4

- Now plug in the corners:



X	Y	P
0	0	$(10)(0) + (12)(0) = 0$
0	8	$(10)(0) + (12)(8) = 96$
3	7	$(10)(3) + (12)(7) = 114$
6	4	$(10)(6) + (12)(4) = 108$
8	1	$(10)(8) + (12)(1) = 92$
8.3	0	$(10)(8.3) + (12)(0) = 83$

- The maximum profit occurs at the $(x = 3, y = 7)$ strategy, with 114 as the profit.

Practice exam #5: Two dice rolling

Two fair dice are rolled.

- What is the probability that the first die is odd?
- What is the probability that the total roll is 9 or larger?
- What is the probability that both the total roll is 9 or larger and the first die rolled was odd?
- What is the probability that the total roll is 9 or larger given that the first die rolled was odd?
- Are the events "total roll is 9 or larger" and "the first die is odd" independent, mutually exclusive, both, or neither?

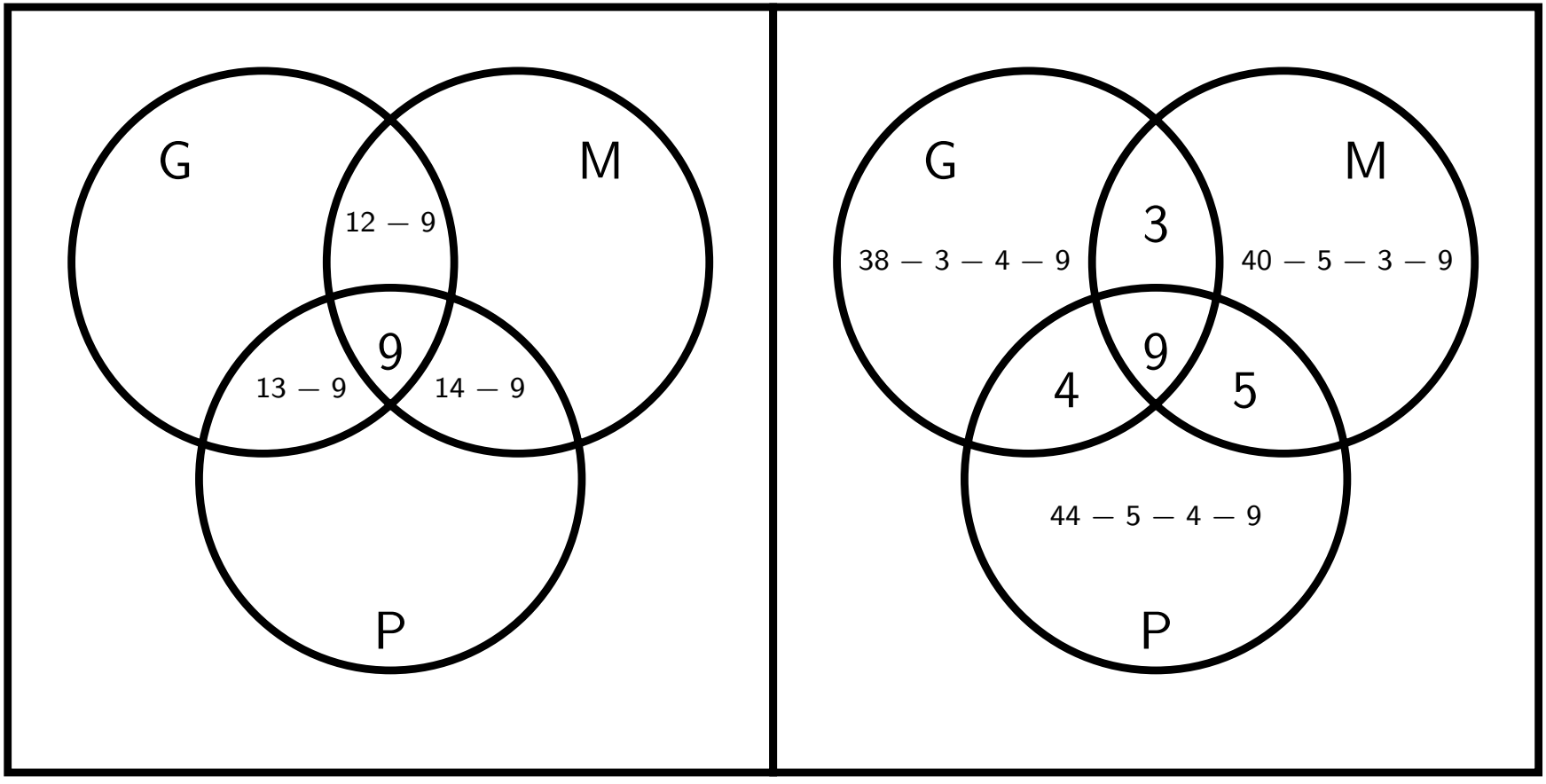
Practice exam #6: inclusion-exclusion

- A survey of 100 College students were asked for their opinions about pizza. They were specifically whether they liked pepperoni, mushrooms, and garlic.
  - 44 students liked pepperoni.
  - 40 students liked mushrooms.
  - 38 students liked garlic.
  - 14 students liked both pepperoni and mushrooms.
  - 13 students liked both pepperoni and garlic.
  - 12 students liked both mushrooms and garlic.
  - 9 students liked all three toppings.

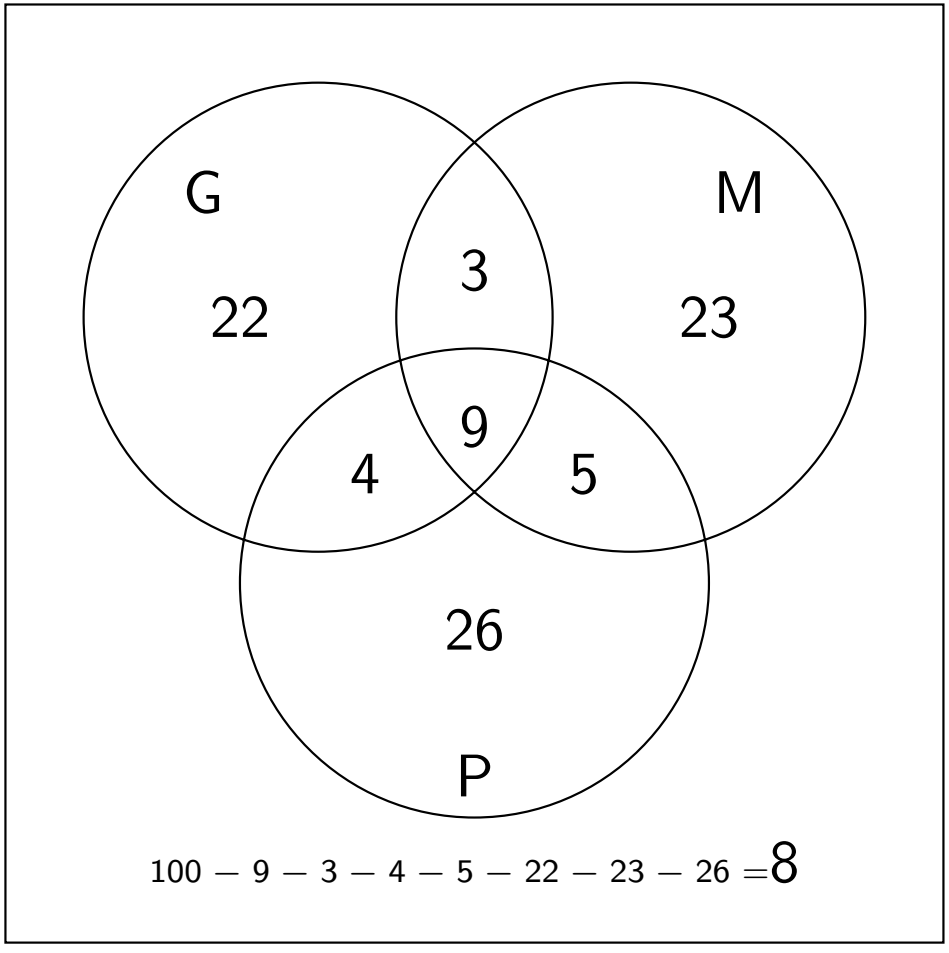
  Based on the above information, answer the following questions. You must show your calculations to receive credit.
- What is the probability that a random student did not like any of the toppings?
- What is the probability that a random student liked at least two of the toppings?

Practice exam #6: solution 1

- Fill in the Venn diagram



Practice exam #6: solution 2



Practice exam #7: Budget cut bias

You are examining a budget cut proposal. In the cut, 85 out of 340 managers will be laid off. A total of 230 out of 940 employees will be laid off, including the managers.

- What is the probability a random employee will be laid off?
- What is the probability a random non-manager will be laid off?
- What is the probability an employee that gets laid off is a manager?
- Are the events "getting laid off" and "being a manager" independent?

Practice exam #7: solution

- Fill in the table form the problem:

	Mngr	Nonm	Employee
Laid Off	85		230
Kept			
Total	340		940

Then you know the margins are just the totals, so you can solve for most of the missing entries.

	Mngr	Nonm	Employee
Laid Off	85	230-85	230
Kept	340-85		940-230
Total	340	940-340	940

The middle entry is both $710 - 255$ and $600 - 145$.

	Mngr	Nonm	Employee
Laid Off	85	145	230
Kept	255	455	710
Total	340	600	940