DEPARTMENT OF MATHEMATICS

Ma 162 Third Exam November 15, 2010

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations. You must show your work to receive credit.
Put your answers in the answer boxes provided, and show your work.
If your answer is not in the box or if you have no work to support your answer, you will receive no credit.
The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	16	
7	12	
8	12	
Total	100	

Please fill in the information below.

NAME: _____ **Section:** _____
Last four digits of Student ID: _____

1. Simple Interest: $I = Prt$. Accumulation: $A = P(1 + rt)$.
2. Compound Interest Accumulation: $A = P(1 + i)^n$. Present value: $P = A(1 + i)^{(-n)}$.
3. Effective rate: $r_{eff} = (1 + \frac{r}{m})^m - 1$.
4. Annuity: Sum: $S = R\frac{((1+i)^n - 1)}{i}$. Present value: $P = R\frac{(1-(1+i)^{-n})}{i}$.
5. Set counting: Two sets: $n(A \bigcup B) = n(A) + n(B) - n(A \bigcap B)$ Three sets: $n(A \bigcup B \bigcup C) = n(A) + n(B) + n(C) - n(A \bigcap B) - n(B \bigcap C) - n(C \bigcap A) + n(A \bigcap B \bigcap C)$.

1. Suppose that P, Q and R are sets with 41 , 42 and 40 members respectively. Calculate the indicated quantities. Display correct formulas or appropriate Venn diagrams.

   i) If $P \bigcup Q$ has 64 members, then $P \bigcap Q$ has $\boxed{}$ members.

   ii) If it is further known that $P \bigcap R$ has 19 members, then $P \bigcup R$ has $\boxed{}$ members.

   iii) If, in addition, $Q - R$ has 22 members, then $Q \bigcap R$ has $\boxed{}$ members.

   iv) Finally, if we are given that the intersection of all three sets P, Q, and R has 12 members, then the union of all three sets has $\boxed{}$ members.

2. Sarah believes that eventually she would need some new machinery for her factory. She expects that it will cost $\$30,000$ after about 5 years from now. She has some money already saved, but notices that it is not earning much interest. She also does not want to risk investing in the stock market.

   So she decides to invest in a fund which guarantees higher returns and would be available exactly after 5 years.

   a) How much would she have to invest now at 8% annual rate compounded annually to have that sum available? [ ]

   Be sure to show the formula used.

   b) Suppose she finds a better investment opportunity which offers **a monthly** compounding at the same annual rate. Assuming 12 nominal months in the year, how much would she need to invest in the new scheme? [ ]

   Be sure to show the formula used.

3. You are about to finance the purchase of a new house with a 15 year loan of 107 thousand dollars at 4.6% APR compounded monthly.

   You are supposed to pay monthly. What is your monthly payment and what is your total of the payments over the period of 15 years?

   Answer. Monthly Payment: ☐ dollars.

   Total of payments: ☐ dollars.

   Be sure to show the formula used.

   b) Suppose that another lender has offered the same loan with the annual interest rate **reduced by half a percent** but for 30 years instead of 15 years. What will be the new monthly payment and the total of the payments over the period of 30 years?

   Answer: Payment: ☐ dollars.

   Total of payments: ☐ dollars.

4. Patrick is fired up about the Green Revolution and uses public transportation. However, he wants to buy a new hybrid car and does not want to take the responsibility of a new car loan. Therefore, he starts putting money into a savings account so he can buy a car after 3 years.

   Patrick figures that he would need to have $30,000 saved, and he wishes to make monthly payments into an account paying 5% interest compounded monthly. Help him figure out the following.

   Be sure to show the formulas used.

   a) How much should Patrick's monthly payment be? [ ]

   b) At the end of 3 years, Patrick decides to continue using public transportation for another year, in hope of a better car. So, he continues making the same monthly payments.

   How much **additional money** will accumulate into the account? [ ]

5. In this problem, assume a 360 day year. "EasyMoney" will loan you 50% of your paycheck of \$600 for 12 days. After 12 days, you pay back the loan plus an interest of \$42.

   a) Calculate the annual simple interest rate for this service. Answer: ☐ per cent.

   b) Across the street, "Rightaway" will loan you 5% more of your paycheck but charge you an interest of \$44. Is this a lower rate? Explain your answer by comparing the rates. Be sure to show all work.

6. A survey of 100 College students revealed the following:

   - 45 students use Fox News for their news.
   - 40 students use Comedy Central for their news.
   - 37 students use MSNBC for their news.
   - 15 students use both Fox News and Comedy Central for their news.
   - 13 students use both Comedy Central and MSNBC for their news.
   - 11 students use both Fox News and MSNBC for their news.
   - 8 students use all three for their news.

   Based on the above information, answer the following questions. You must show your calculations to receive credit.

   (a)How many students surveyed use none of the three news sources?

   Answer: [ ]

   (b)How many students surveyed use at least two of the three news sources?

   Answer: [ ]

7. Cal wants to generate a random password automatically. He first chooses 4 letters from the usual alphabet, with repetition allowed. Then he chooses 4 integers in the interval $[5, 11]$ with repetition allowed.

   Then the password is simply composed by writing the alphabet sequence followed by the number sequence.

   a) How many different passwords does his scheme create? **Answer:** [ ]

   b) Suppose that Cal discards passwords where all the letters are the same, then **how many different passwords** are possible? **Answer** [ ]

   Suggestion: It is recommended to write a correct formula, without multiplying out.

8. There are 19 students in a class of graduate students. A delegation is to be chosen to join a field trip on an external research project. The delegation shall have a leader, a spokesman, a secretary and a researcher.

   Answer the following questions. Be sure to show your reasoning. Just numerical answers shall earn no credit.

   (a) How many different delegations can be formed from the students in the class?

   Answer:

   (b) Suppose that Matt is chosen as the spokesman, since he is already the student representative on the school council. How many different delegations can now be formed?

   Answer:

1 Answer Key for exam3v-5

1. ◇ (i) 19
   ◇ (ii) 62
   ◇ (iii) 20
   ◇ (iv) 77

2. ◇ a) $20,417.50
   ◇ b) $20,136.31

3. ◇ a) Monthly Payment $ 824.02 Payout $ 148,323.95
   ◇ b)517.02 Total 186,128.00

4. ◇ a) Monthly saving $774.13
   ◇ Additional accumulation $11,040.25

5. ◇ a)420.00per cent
   ◇ b)400.00per cent Better? yes!

6. ◇ (a) 9
   ◇ (b) 23

7. ◇ a) 1097199376 passwords. (26 raised to 4) * (7 raised to 4).
   ◇ b) 1097136950 passwords. (26 raised to 4 minus 26) * (7 raised to 4).

8. ◇ (a) 93024
   ◇ (b) 4896