

MA162: Finite mathematics

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January 25, 2012

SCHEDULE:

- HW 2.1-2.2 are due Friday, Jan 27th, 2012.
- HW 2.3-2.4 are due Friday, Feb 3rd, 2012.
- Exam 1 is Monday, Feb 6th, 5:00pm-7:00pm in CB106 and CB118.

Today we will cover 2.2, augmented matrices, and the elimination algorithm

Start the ZESTY problem on the worksheet now!

2.2: Do we already know this?

- You and the crew have lunch at Fried-ees most days
- Day 1: you got the Zesty meal for \$5
- Day 2: You and a pal got the Yummy bunch, and your apprentice got the Zesty; one check for \$17
- Day 3: Your pal got the Xtra crispy, your apprentice got the Yummy, and you got the Zesty for \$18
- How much does the Xtra, the Yummy, and the Zesty each cost?

2.2: Do it with equations

$$X + Y + Z = 18$$

$$2Y + Z = 17$$

$$Z = 5$$

- If $Z = 5$, then we know Z :

$$X + Y + 5 = 18$$

$$2Y + 5 = 17$$

- If $2y + 5 = 17$, then $2y = 12$ and $y = 6$

$$X + 6 + 5 = 18$$

- If $x + 6 + 5 = 18$, then $x = 7$.

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- If $x + 6 + 5 = 18$, then $x = 7$.

2.2: Efficiently solving systems

- We solved systems last time with two variables
- Real decisions involve balancing half a dozen variables
- Two main changes to handle this:
 - **Write down less** to see the important parts clearly
 - Use a **systematic** method to solve

2.2: Efficient notation

- We worked some equations with the variables x, y
- We could have used M and T
- The letters we used did not matter; just placeholders
- Why do we even write them down?
- The plus signs and equals are pretty boring too.
- The only part we need are the numbers
(and where the numbers are)

2.2: Augmented matrices

$$x + 2y = 4$$

$$y + 5z = 7$$

$$8x + 17y - z = 9$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right]$$

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2.2: Augmented matrices

$$x + 2y = 4$$

$$y + 5z = 7$$

$$8x + 17y - z = 9$$

$$1x + 2y = 4$$

$$1y + 5z = 7$$

$$8x + 17y - 1z = 9$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right]$$

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$$x + 2y = 4$$

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$$8x + 17y - z = 9$$

$$1x + 2y + 0z = 4$$

$$0x + 1y + 5z = 7$$

$$8x + 17y + -1z = 9$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right]$$

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2.2: More examples

$$\begin{array}{rcl} 2x + 3z = 4 & 2x & + 0y & + 3z & = & 4 \\ 6z + 5y = 7 & 0x & + 5y & + 6z & = & 7 \\ 8x + 9y = 1 & 8x & + 9y & + 0z & = & 1 \end{array} \quad \left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 8 & 9 & 0 & 1 \end{array} \right]$$

$$\begin{array}{rcl} 4x + 3z = 2 & 4x & + 0y & + 3z & = & 2 \\ 8z - y = 7 & 0x & - 1y & + 8z & = & 7 \\ 5x - 9y = 6 & 5x & - 9y & + 0z & = & 6 \end{array} \quad \left[\begin{array}{ccc|c} 4 & 0 & 3 & 2 \\ 0 & -1 & 8 & 7 \\ 5 & -9 & 0 & 6 \end{array} \right]$$

$$\begin{array}{rcl} y = 3 - 2x & 2x & + 1y & + 0z & = & 3 \\ z = 7 + 4y & 0x & - 4y & + 1z & = & 7 \\ x = 6 + 5z & x & + 0y & - 5z & = & 6 \end{array} \quad \left[\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & -4 & 1 & 7 \\ 1 & 0 & -5 & 6 \end{array} \right]$$

2.2: Efficient notation

- We now have a very clean way to write down systems of equations
- Make sure you can convert from a system of equations to the **augmented matrix**
- Make sure you can convert from an augmented matrix to a system of equations

2.2: A systematic procedure

- Now we will learn a method of solving systems
- We will transform the equations until they look like (REF):

$$\begin{aligned}x + 2y + 3z &= 4 \\5y + 6z &= 7 \\8z &= 9\end{aligned}$$

- Next time, we will transform them until they look like (RREF):

$$\begin{aligned}x &= 1 \\y &= 2 \\z &= 3\end{aligned}$$

- We will do this by following a set of rules
- Your work on the exam is graded **strictly**

2.2: First step: Find pivots

- The 0th step is to make sure you have got an augmented matrix
- Once you do we look for **pivots**
- Each row should have a pivot;
it is the **first nonzero** number in the row

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right]$$

- We want **one pivot per column**
- We are usually **disappointed**

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2.2: Second step: Choose target

- If there are two pivots in one column, we **eliminate** one of them
- The **active pivot** is the first pivot in the first bad column
- The **target pivot** is the next pivot in the first bad column

The diagram shows a matrix with a vertical bar on the right side, representing an augmented matrix. The matrix is:

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right]$$

A blue arrow labeled "Active" points to the circled '1' in the first row, first column. A red arrow labeled "Target" points to the circled '8' in the third row, first column.

- We want to ZERO out the **target** by subtracting a multiple of the **active**

2.2: Third step: Eliminate the target

- We are now going to subtract a multiple of the **active row** from the **target row**
- We choose the multiple: $\frac{\text{target pivot}}{\text{active pivot}}$
- In our example, we choose $\frac{8}{1} = 8$

$$\begin{array}{r} R_3 \\ -8R_1 \\ \hline \text{New } R_3 \end{array} \quad \begin{array}{cccc} 8 & 17 & -1 & 9 \\ 1 & 2 & 0 & 4 \end{array}$$

$$\begin{array}{cccc} 8 & 17 & -1 & 9 \\ 0 & -16 & -8 & -32 \\ \hline 0 & 1 & -1 & -23 \end{array}$$

- We changed the old **8** to a zero!
- This **new row** will replace our old target row

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$$\begin{array}{r} R_3 \\ -8R_1 \end{array} \quad \begin{array}{r} 8 \quad 17 \quad -1 \quad 9 \\ -8 \cdot (\quad 1 \quad 2 \quad 0 \quad 4) \end{array} \quad \begin{array}{r} + \\ -8 \end{array} \quad \begin{array}{r} 8 \quad 17 \quad -1 \quad 9 \\ -8 \quad -16 \quad 0 \quad -32 \\ \hline 0 \quad 1 \quad -1 \quad -23 \end{array}$$

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2.2: Fourth step: regroup

- Now we rewrite our new matrix and start over with an **easier** system

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 1 & -1 & -23 \end{array} \right]$$

- We also need to **show our work** in a **very specific way**

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$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 8 & 17 & -1 & 9 \end{array} \right] \xrightarrow{R_3 - 8R_1} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 1 & -1 & -23 \end{array} \right]$$

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- Now we begin again with our new **simpler** system:

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 1 & -1 & -23 \end{array} \right]$$

- We find the pivots
- Second column has pivots in rows 2 and 3, so need to clear again!
- This time you do it! (On worksheet)

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Brief version of 2-4 (again)

$$\begin{aligned} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 1 & -1 & -23 \end{array} \right] & \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & -6 & -30 \end{array} \right] \\ & \xrightarrow[\text{(optional)}]{R_3 / (-6)} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 5 \end{array} \right] \end{aligned}$$

- The pivots are in a nice diagonal
- No column has more than one pivot, so **REF**
- We can solve this using algebra, first for z , then for y , then for x

2.2: Final step – Back substitution

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & -6 & -30 \end{array} \right] \quad \begin{cases} 1X + 2Y + 0Z = 4 \\ 0X + 1Y + 5Z = 7 \\ 0X + 0Y + -6Z = -30 \end{cases}$$

- $-6Z = -30$ is better known as $Z = 5$
- $Y + 5Z = 7$ is better known as $Y + 25 = 7$, or $Y = -18$ to its friends
- $X + 2Y = 4$ is better known as $X - 36 = 4$, so $X = 40$.
- $(X = 40, Y = -18, Z = 5)$

2.2: Real question

- You have three types of workers: packers, sewers, cutters.
- You have three types of products: short-sleeve, sleeveless, long-sleeve.
- It takes the following amount of time to make them:

	Short	Less	Long
Pack	4	3	4
Sew	24	22	28
Cut	12	9	15

- You have 24 hours of packers, 80 hours of cutters, and 160 hours of sewers
- How many of each should you make to keep everyone working?

2.2: As system, as matrix

- As a system of equations:

Make x short-sleeve, y sleeveless, z long-sleeve

$$\begin{cases} 4x + 3y + 4z = 1440 \\ 24x + 22y + 28z = 9600 \\ 12x + 9y + 15z = 4800 \end{cases}$$

- As a matrix:

$$\left(\begin{array}{ccc|c} 4 & 3 & 4 & 1440 \\ 24 & 22 & 28 & 9600 \\ 12 & 9 & 15 & 4800 \end{array} \right)$$

2.2: REF it

$$\left(\begin{array}{ccc|c} 4 & 3 & 4 & 1440 \\ 24 & 22 & 28 & 9600 \\ 12 & 9 & 15 & 4800 \end{array} \right) \xrightarrow[\substack{R_2-3R_1 \\ R_3-6R_1}]{R_2-3R_1} \left(\begin{array}{ccc|c} 4 & 3 & 4 & 1440 \\ 0 & 4 & 4 & 960 \\ 0 & 0 & 3 & 480 \end{array} \right) \quad \text{REF}$$

- As equations:
$$\begin{cases} 4x + 3y + 4z = 1440 \\ 4y + 4z = 960 \\ 3z = 480 \end{cases}$$
- $z = 480/3 = 160$, then $4y + 4(160) = 960$ and $y = 80$, then ... and $x = 140$
- So make 140 short-sleeve, 80 sleeveless, and 160 long-sleeves