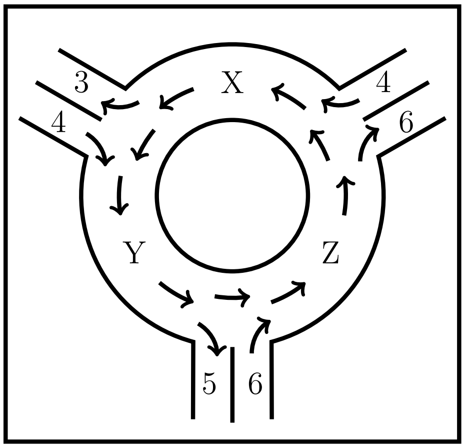

Which is busier, X, Y, or Z?

(a) Write equations for the conservation of cars at each of the intersections.

(b) Write the equations as a matrix.

(c) Row reduce the matrix as far as you can, and circle the pivots in the final matrix

(d) Write down the equations corresponding to the REF matrix, and solve them for the circled variables from part (c)

(e) Even if we don't know what Y is, can we tell which road is busiest?

(f) Give a possible set of values for ($X =$ ___, $Y =$ ___, $Z =$ ___)

(g) Give a different possible set of values for ($X =$ ___, $Y =$ ___, $Z =$ ___)

(h) Assuming people don't go the wrong way around the round-about (including trying to make immediate U-turns), what is the least busy each section of road could be?

Keep going! Apply row operations to get even more zeros:

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 15 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{R_2-R_3} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 15 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{R_1-R_3} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Now the equations read: $1x = 3$, $1y = 4$, and $1z = 5$.

Now you try some examples:

(a) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 59 \\ 0 & 1 & 5 & 47 \\ 0 & 0 & 1 & 8 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 16 \\ 0 & 1 & -4 & -25 \\ 0 & 0 & 1 & 8 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

REFs with more variables than pivots Sometimes there are more variables than pivots. Columns that have exactly one pivot tell you how to solve for a variable. Columns with no pivots cannot be solved.

Consider this matrix:

X	Y	Z	T	RHS
1	7	0	8	9
0	0	1	4	5
0	0	0	0	0

(a) What rows can you use to zero out the 7?

(b) What rows can you use to zero out the 4? What becomes non-zero if you do this?

(c) What rows can you use to zero out the 8? What becomes non-zero if you do this?

Consider the equations $X = 9 - 7Y - 8T$ and $Z = 5 - 4T$.

(a) If $Y = 0$ and $T = 0$, solve for all the variables: $(X = \_\_\_, Y = \_\_\_, Z = \_\_\_, T = \_\_\_)$

(b) If $Y = 0$ and $T = 1$, solve for all the variables: $(X = \_\_\_, Y = \_\_\_, Z = \_\_\_, T = \_\_\_)$

Whatever someone decides to make Y and T, we can always find X and Z. Y and T are called **FREE** since they are free to be whatever.

RREF elimination Repeat until done: Find a pivot. Make it 1 by dividing the whole pivot row. Use balancing row ops to get zeros above and below the pivot. An example is given by exercise 4.1.4:

$$\begin{array}{rrrrr|r} H & A & P & U & B & RHS \\ \hline 1 & 1 & 0 & 1 & 0 & 20 \\ -1 & 4 & 0 & 0 & 1 & 0 \\ -10 & -12 & 1 & 0 & 0 & 0 \end{array} \xrightarrow[R_3+10R_1]{R_2+R_1} \begin{array}{rrrrr|r} H & A & P & U & B & RHS \\ \hline 1 & 1 & 0 & 1 & 0 & 20 \\ 0 & 5 & 0 & 1 & 1 & 20 \\ 0 & -2 & 1 & 10 & 0 & 200 \end{array} \xrightarrow[R_3+\frac{2}{5}R_2]{R_1-\frac{1}{5}R_2 \atop R_2/5} \begin{array}{rrrrr|r} H & A & P & U & B & RHS \\ \hline 1 & 0 & 0 & 0.8 & -0.2 & 16 \\ 0 & 1 & 0 & 0.2 & 0.2 & 4 \\ 0 & 0 & 1 & 10.4 & 0.4 & 200 \end{array}$$

The previous word problem asked how much one should invest in home or auto insurance policies. H is how much to invest in home, A is how much to invest in auto, U is how much remains uninvested, B is how safe the investment is, measured as $B = H - 4A$ since one thinks that $H \geq 4A$ is a good idea. P is profit. The final matrix gives the equations

$$\begin{array}{rl} H+\ 0.8U-0.2B = 16 & \\ A+\ 0.2U+0.2B = 4 & \rightarrow \\ P+10.4U+0.4B = 200 & \end{array} \quad \begin{array}{rl} H = & 16-\ 0.8U+0.2B \\ A = & 4-\ 0.2U-0.2B \\ P = & 200-10.4U-0.4B \end{array}$$

These might look a little complicated, but it turns out to be easy to decide what to make U and B. If we leave money uninvested, we don't make as much profit. Every dollar we don't invest costs us \$10.40 in profit since $P - \$200 - 10.4U \ldots$. The obvious choice is $U = 0$; invest it all. Similarly if we don't invest as aggressively as we are allowed, then we cannot get the largest return. Every dollar we use to be safe costs us \$0.40 in profit since $P = \$200 - 0.4B - \ldots$. The obvious choice is $B = 0$.

This gives us: $H = 16$, $A = 4$, $P = 200$ as our best solution, though of course there are many other solutions with lower profit.