

DEPARTMENT OF MATHEMATICS

Ma 162 First Exam February 6, 2012

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	11	
2	11	
3	11	
4	11	
5	11	
6	11	
7	11	
8	11	
9	11	
Total	100	

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(Practice version)

1. A courier travels from city Ashton with coordinates $(0, 0)$ to city Cranston with coordinates $(125, 135)$. He must pass through **exactly one of the cities** Brady with coordinates $(72, 45)$ or Dalton $(45, 72)$ along the way. Assume he travels a straight line between cities.
- (a) Which city should he pass through (Brady or Dalton) in order to minimize his trip distance from Ashton to Cranston?

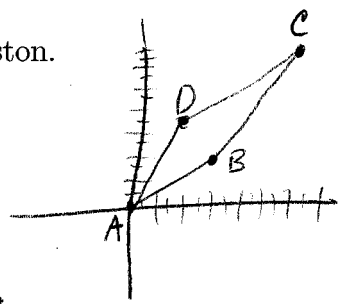
He should pass through city Dalton on his way to Cranston.

$$AB = \sqrt{72^2 + 45^2} = \sqrt{7209} = 84.91$$

$$BC = \sqrt{(125-72)^2 + (135-45)^2} = \sqrt{53^2 + 90^2} = 104.45$$

$$\overset{\text{vs}}{AD} = \sqrt{45^2 + 72^2} = \sqrt{7209} = 84.91$$

$$DC = \sqrt{(125-45)^2 + (135-72)^2} = \sqrt{80^2 + 63^2} = 101.83$$



$AB = AD$ but $DC < BC$ so $A-D-C < A-B-C$

- (b) What is the total minimum length of his trip from Ashton to Cranston, taking into account the stop in the city from part (a)?

Minimum trip length is: 186.73

$$84.91 + 101.83$$

Make sure to show your work. Answers with no work receive no credit.

2. Point A has coordinates (7, 3), and point B has coordinates (0, 5).

(a) What is the distance from A to B and what is the slope of the line joining A to B?

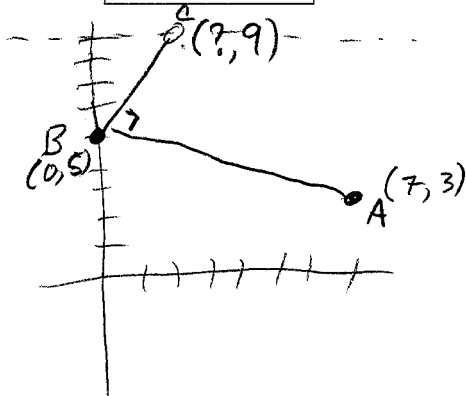
distance: $\sqrt{53}$, slope: $-2/7$

$$AB = \sqrt{(7-0)^2 + (3-5)^2} = \sqrt{7^2 + 2^2} = \sqrt{53} \approx 7.28$$

$$m_{AB} = (3-5)/(7-0) = -2/7$$

(b) Suppose that the point C with coordinates (x, 9) is such that the triangle ABC is a right triangle with right angle at B. Determine the value of x. (Note: The coordinates of A and B were given at the top of the problem.)

x = $\frac{8}{7}$



$AB \perp BC$ are perpendicular

$$\text{so } m_{BC} = \frac{-1}{m_{AB}} = \frac{-1}{(-2/7)} = 7/2$$

$$\text{but } m_{BC} = \frac{5-9}{0-x}$$

$$\text{so } \frac{-4}{-x} = \frac{7}{2} \quad 7x = 8$$

$x = \frac{8}{7}$ (and this is believable based on picture)

Make sure to show your work. Answers with no work receive no credit.

3. The Flörgerstrøm company makes valve cleaning units for flügelhorns. The cost function for their manufacturing line is $C = 2x + 3500$, where x is the number of VCUs produced per month and C is measured in dollars. The company expects \$7 in revenue per unit.

(a) Determine the linear profit function for the Flörgerstrøm company in the usual form $P = mx + b$, assuming they can sell all the units they manufacture.

$$P = \boxed{5x - 3500}$$

$$R = 7x$$

$$C = 2x + 3500$$

$$P = R - C = 5x - 3500$$

(b) Determine the break-even value for x and the break-even cost C at that value for x .

$$x = \boxed{700 \text{ units}}$$

$$C = \boxed{\$4900}$$

$$R = C$$

$$7x = 2x + 3500$$

$$5x = 3500$$

$$x = 700$$

$$C = 2(700) + 3500 = 4900$$

$$R = 7(700) = 4900 \checkmark$$

Make sure to show your work. Answers with no work receive no credit.

4. In a free market, the supply equation for a supplier of corn is $x = 36p + 200$ where the price p is in dollars and x is in bushels. When the price is \$4 per bushel the demand is 1170 bushels. When the price goes up to \$17 per bushel the demand drops to 0 bushels. Assuming that the demand equation is also linear, find the equilibrium price and the number of bushels supplied at that equilibrium price.

Demand equation:

$$x = -90p + 1530$$

$$p = \$10.56$$

$$x = 580 \text{ bushels}$$

Demand = ? p + ? solve for ?

$$x = Ap + B$$

$$1170 = A(4) + B$$

$$0 = A(17) + B$$

$$\begin{array}{r} 1170 = -13A + 0 \end{array}$$

$$A = 1170 / -13 = -90$$

$$0 = (-90)(17) + B$$

$$B = (90)(17) = 1530$$

Equilibrium is supply = demand

$$36p + 200 = -90p + 1530$$

$$126p = 1330$$

$$p = \frac{1330}{126} \approx \$10.56$$

$$x = 36\left(\frac{1330}{126}\right) + 200 = 580 \text{ bushels}$$

Make sure to express yourself clearly. X=DOG receives no credit.

5. For the following word problem: (a) Write down variables describing the (numerical) business decision to be made, (b) write down equations that constrain your decision, (c) convert the equations to an augmented matrix. **You need not solve the system.**

Mr. Marjoram is renting an automated stuffed animal factory with three machines: a sewing machine, a stuffing machine, and a trimming machine. He has programmed it to make Pandas, Dogs, and Birds, but some of the animals take longer on some of the machines, so he isn't sure how many of each animal to make. He wants the machines to be in constant use (so he feels he got his money's worth; why pay for an idle machine). The production times and available times are given in the table below. How many of each animal should he make?

	Sewing	Stuffing	Trimming
Panda	12 min per	13 min per	14 min per
Dog	16 min per	17 min per	15 min per
Bird	20 min per	18 min per	19 min per
Available	12 hours 720 min	12 hours	12 hours

The variables describing the decision are:

$x = \#$ of pandas to make

$y = \#$ of dogs to make

$z = \#$ of birds to make

The equations to be solved are:

$$\begin{array}{l} \text{Keep Sewing Machine Busy: } 12x + 16y + 20z = 720 \\ \text{" Stuffing " " : } 13x + 17y + 18z = 720 \\ \text{" Trimming " " : } 14x + 15y + 19z = 720 \end{array}$$

The augmented matrix describing the equations is:

$$\begin{array}{l} \text{Sew} \\ \text{Stuff} \\ \text{Trim} \end{array} \begin{bmatrix} \text{Panda} & \text{Dog} & \text{Bird} & \text{Avail} \\ 12 & 16 & 20 & 720 \\ 13 & 17 & 18 & 720 \\ 14 & 15 & 19 & 720 \end{bmatrix}$$

Make sure to show your work. Answers with no work receive no credit.

6. Here is the augmented matrix of a linear system of equations. Take this matrix to RREF. Be sure to label your reduction operations in standard notation. You need not solve for the variables.

$$\left(\begin{array}{cccc|c} x & y & z & w & \text{RHS} \\ 7 & 6 & 5 & 4 & 3 \\ 0 & 3 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right) \begin{array}{l} R_1 - 5R_3 \\ R_2 - 4R_3 \end{array} \rightarrow \left(\begin{array}{cccc|c} 7 & 6 & 0 & -6 & -12 \\ 0 & 3 & 0 & -3 & -6 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right)$$

$$\xrightarrow{R_1 - 2R_2} \left(\begin{array}{cccc|c} 7 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & -6 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right)$$

$$\begin{array}{l} R_1 / 7 \\ R_2 / 3 \end{array} \rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right) \begin{array}{l} \text{RREF} \\ \text{Done!} \end{array}$$

(#7 asks!)

$$\begin{array}{l} x = 0 \\ y - w = -2 \\ z + 2w = 3 \end{array} \quad \text{so} \quad \begin{array}{l} x = 0 \\ y = w - 2 \\ z = 3 - 2w \\ w \text{ is Free} \end{array}$$

Make sure to show your work. Answers with no work receive no credit.

7. Here is the augmented matrix of a linear system of equations. As usual, the variables are mentioned for your convenience.

$$\left(\begin{array}{cccc|c} x & y & z & w & \text{RHS} \\ \hline 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{array} \right)$$

- (a) Is this matrix in REF or RREF or neither of these?

Both REF and RREF

- (b) Finish the solution process as needed and determine the complete solution of the system by filling in the answers below. If a variable is free, then enter the word "free" as its value. Be sure to show all calculations.

$$x = \boxed{3 - 2y}$$

$$1x + 2y + 0z + 0w = 3$$

$$x + 2y = 3$$

$$y = \boxed{\text{FREE}}$$

$$z = \boxed{4}$$

$$w = \boxed{5}$$

Make sure to show your work. Answers with no work receive no credit.

8. Your friend's calculator has never been the same since it went swimming last year. He asked it to solve Mr. Marjoram's problem, or at least bring the matrix to REF. It claims the answer is:

$$\begin{array}{c}
 \times \quad \checkmark \quad \checkmark \\
 \left[\begin{array}{ccc|c}
 P & D & B & \text{RHS} \\
 3 & 4 & 5 & 180 \\
 0 & 1 & 11 & 180 \\
 0 & 0 & 1 & 15
 \end{array} \right]
 \end{array}
 \left\{ \begin{array}{l}
 3P = 45, P = 15 \\
 3P + (4)(15) + (5)(15) = 180, 3P = 180 - 75 - 60 \\
 D + (15)(11) = 180, D = 180 - 165 = 15 \\
 B = 15
 \end{array} \right.$$

- (a) Assume the calculator is right, and solve for P , D , and B .

$$\boxed{(P = \underline{15}, D = \underline{15}, B = \underline{15})}$$

- (b) Is your answer in part (a) actually right? Check your constraints in problem #5, and show that the production goals do or do not keep the machines busy.

$$\begin{array}{l}
 \text{Sewing Machine is used } 12(\overset{P}{15}) + 16(\overset{D}{15}) + 20(\overset{B}{15}) = 720 \text{ min} = 12 \text{ hrs } \checkmark \\
 \text{Stuffing Machine is used } 13(15) + 17(15) + 18(15) = 720 \text{ min} = 12 \text{ hrs } \checkmark \\
 \text{Trimming Machine is used } 14(15) + 15(15) + 14(15) = 720 \text{ min} = 12 \text{ hrs } \checkmark
 \end{array}$$

Your other friend's calculator hasn't been the same since it spent the night in the chemistry lab. It says the REF is:

$$\begin{array}{l}
 P = 210/14 = 15 \\
 14P = 720 - 510 \\
 14P + 15(15) + 19(15) = 720
 \end{array}
 \left[\begin{array}{ccc|c}
 P & D & B & \text{RHS} \\
 14 & 15 & 19 & 720 \\
 0 & 11 & 13 & 360 \\
 0 & 0 & 1 & 15
 \end{array} \right]
 \left\{ \begin{array}{l}
 11D = 165, D = 15 \\
 11D = 360 - 195 \\
 11D + 13(15) = 360 \\
 B = 15
 \end{array} \right.$$

- (c) Assume this calculator is also right, somehow. Solve for P , D , and B .

$$\boxed{(P = \underline{15}, D = \underline{15}, B = \underline{15})}$$

Same as (a), so also right!

Make sure to show your work. Answers with no work receive no credit.

9. A farmer is in the corn and soy business. Every acre of corn will cost him \$40 and every acre of soy will cost him \$32. He has 100 acres and a budget of \$3,480. How much should he plant of each crop in order to use up all the land and all the budget?

He should plant 35 acres of corn and 65 acres of soy.

This will use 100 acres of land and \$ 3480 from the budget.

$C = \#$ of acres of Corn

$S = \#$ of acres of Soy

$$C + S = 100 \quad (\text{use all land})$$

$$40C + 32S = 3480 \quad (\text{use all money})$$

$$\left(\begin{array}{cc|c} 1 & 1 & 100 \\ 40 & 32 & 3480 \end{array} \right) \xrightarrow{R_2 - 40R_1} \left(\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & -8 & -520 \end{array} \right) \xrightarrow{R_2 / -8} \left(\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & 1 & 65 \end{array} \right)$$

$$\xrightarrow{R_1 - R_2} \left(\begin{array}{cc|c} C & S & \text{RHS} \\ 1 & 0 & 35 \\ 0 & 1 & 65 \end{array} \right) \text{ RREF} \quad \begin{array}{l} C = 35 \\ S = 65 \end{array}$$

$$C + S \text{ is } 35 + 65 = 100$$

$$40C + 32S \text{ is } 1400 + 2080 = 3480$$