Proofread.
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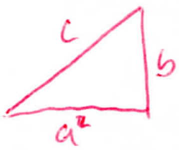
1. A courier travels from city Ashton with coordinates $(0,0)$ to city Cranston with coordinates $(125,135)$. He must pass through **exactly one of the cities** Brady with coordinates $(72,45)$ or Dalton $(45,72)$ along the way. Assume he travels a straight line between cities.

   (a) Which city should he pass through (Brady or Dalton) in order to minimize his trip distance from Ashton to Cranston?

   He should pass through city $\boxed{\text{Dalton}}$ on his way to Cranston.



$a^2+b^2=c^2$

$72^2+45^2=45^2+72^2$ so distance from A to D and A to B is the same

Distance from D to C $(125-45)^2+(135-72)^2 = 80^2+63^2 = 6400+3969$
$= 10,369$

from B to C $(125-72)^2+(135-45)^2 = 53^2+90^2 = 2,809+8,100$
$= 10,909.$

10,909 > 10,369
so Dalton is the shorter Route.

   (b) What is the total minimum length of his trip from Ashton to Cranston, taking into account the stop in the city from part (a)?

   Minimum trip length is: $\boxed{186.74 \text{ miles}}$

$\sqrt{45^2+72^2}+\sqrt{10,369}$

$\sqrt{2025+5184}+\sqrt{10,369}$

$\sqrt{7209}+\sqrt{10,369} \approx 84.91+101.83 = 186.74$ miles

2. Point A has coordinates $(7,3)$,and point B has coordinates $(0,5)$.

(a) What is the distance from A to B and what is the slope of the line joining A to B?

distance: $\boxed{\sqrt{53}}$, slope: $\boxed{-\frac{2}{7}}$

$\cancel{x_1}$ $(x_1, y_1) = (7,3)$

$(x_2, y_2) = (0,5)$

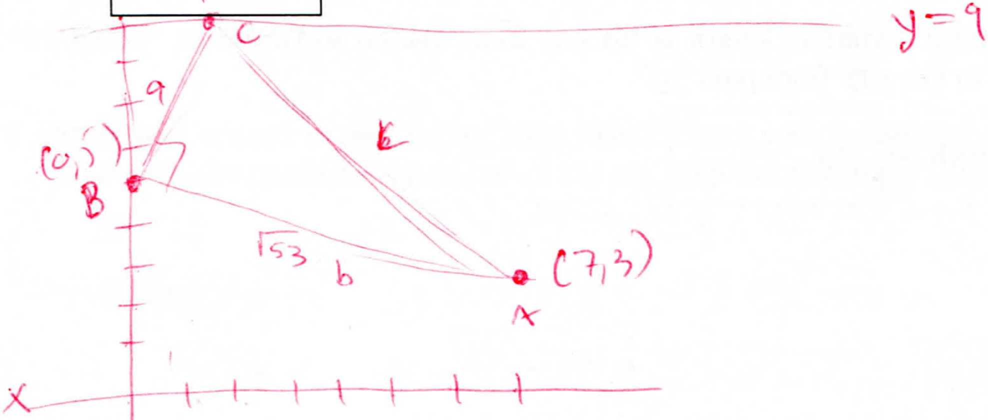
$d = \sqrt{(7-0)^2 + (3-5)^2}$

$= \sqrt{49+4} = \sqrt{53}$

$\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-3}{0-7} = \dfrac{2}{-7}$

(b) Suppose that the point C with coordinates $(x, 9)$ is such that the triangle ABC is a right triangle with right angle at B. Determine the value of x. (Note: The coordinates of A and B were given at the top of the problem.)

$x = \boxed{8/7}$



$AC = \sqrt{(7-x)^2 + (9-3)^2} = \sqrt{(7-x)^2 + 36}$

$BC = \sqrt{(0-x)^2 + (9-5)^2} = \sqrt{x^2 + 16}$

$a^2 + b^2 = c^2$

$\cancel{(7-x)^2 + 36 + 53 =}$

$53 + x^2 + 16 = (7-x)^2 + 36$

$$\begin{array}{r} 53 \\ 16 \\ \hline 69 \\ -36 \\ \hline 33 \end{array}$$

$33 = 49 - 14x + x^2 - x^2$

$14x = 49 - 33 = 16 \qquad x = \dfrac{16}{14} = \dfrac{8}{7}$

3. The Flörgerstrøm company makes valve cleaning units for flügelhorns. The cost function for their manufacturing line is $C = 2x + 3500$, where x is the number of VCUs produced per month and C is measured in dollars. The company expects \$7 in revenue per unit.

(a) Determine the linear profit function for the Flörgerstrøm company in the usual form $P = mx + b$, assuming they can sell all the units they manufacture.

$P = \boxed{5x - 3500}$

$P = R - C$

$P = 7x - (2x + 3500)$

$= 5x - 3500$

(b) Determine the break-even value for x and the break-even cost C at that value for x.

$x = \boxed{1750}$ $C = \boxed{7000}$

$C = R$

$7x = 5x + 3500$

$2x = 3500$

$x = 1750$

$C = 2(1750) + 3500$

$= 3500 + 3500 = 7000$

4. In a free market, the supply equation for a supplier of corn is $x = 36p + 200$ where the price p is in dollars and x is in bushels. When the price is \$4 per bushel the demand is 1170 bushels. When the price goes up to \$17 per bushel the demand drops to 0 bushels. Assuming that the demand equation is also linear, find the equilibrium price and the number of bushels supplied at that equilibrium price.

Demand equation: $\boxed{p = \frac{-13}{1170}x + 17}$

$p = \boxed{\cancel{10.56}\ 10.56}$ $x = \boxed{580}$ bushels

Points $(1170, 4)$ & $(0, 17)$

$x = 36p + 200$

$m_{\text{Demand}} = \frac{17-4}{0-1170} = \frac{-13}{1170}$

$36p = x - 200$

point slope

$p = \frac{1}{36}x - \frac{200}{36}$

$p\ \cancel{y} - 17 = \frac{-13}{1170}(x - 0)$

$p\ \cancel{y} = \frac{-13}{1170}x + 17$

$D = S$

$\left(\frac{-13}{1170}x + 17 = \frac{1}{36}x - \frac{200}{36}\right) \mid 36$

$p = \frac{580}{36} - \frac{200}{36}$

$p = \frac{580 - 200}{36}$

$\left(\frac{-13}{1170} \cdot 36x + 17 \cdot 36 = (x - 200)\right) 1170$

$= \frac{380}{36}$

$-13 \cdot 36x + 17 \cdot 36 \cdot 1170 = 1170x - 200 \cdot 1170$

10.56

$17 \cdot 36 \cdot 1170 + 200 \cdot 1170 = (1170 + 13 \cdot 36)x$

$\frac{17 \cdot 36 \cdot 1170 + 200 \cdot 1170}{1170 + 13 \cdot 36} = x$

$\frac{716040 + 234000}{1638} = x$

$x = \cancel{2323}$ ~~bushels~~ 580

$p = 10.56$

5. For the following word problem: (a) Write down variables describing the (numerical) business decision to be made, (b) write down equations that constrain your decision, (c) convert the equations to an augmented matrix. **You need not solve the system.**

Mr. Marjoram is renting an automated stuffed animal factory with three machines: a sewing machine, a stuffing machine, and a trimming machine. He has programmed it to make Pandas, Dogs, and Birds, but some of the animals take longer on some of the machines, so he isn't sure how many of each animal to make. He wants the machines to be in constant use (so he feels he got his money's worth; why pay for an idle machine). The production times and available times are given in the table below. How many of each animal should he make?

	Sewing	Stuffing	Trimming
Panda	12 min per	13 min per	14 min per
Dog	16 min per	17 min per	15 min per
Bird	20 min per	18 min per	19 min per
Available	12 hours	12 hours	12 hours

The variables describing the decision are:

$P :=$ # of Pandas

$D :=$ # of Dogs

$B :=$ # of Birds

The equations to be solved are:

$$12 \cdot P + 16 \cdot D + 20 \cdot D = 12 \cdot 60 = 720$$

$$13 \cdot P + 17 \cdot D + 18 \cdot B = 12 \cdot 60 = 720$$

$$14 P + 15 \cdot D + 19 \cdot B = 720$$

The augmented matrix describing the equations is:

$$\left[\begin{array}{ccc|c} 12 & 16 & 20 & 720 \\ 13 & 17 & 18 & 720 \\ 14 & 15 & 19 & 720 \end{array}\right]$$

6. Here is the augmented matrix of a linear system of equations. Take this matrix to RREF. Be sure to label your reduction operations in standard notation. You need not solve for the variables.

$$\left(\begin{array}{cccc|c} x & y & z & w & \text{RHS} \\ \hline 7 & 6 & 5 & 4 & 3 \\ 0 & 3 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right) \xrightarrow{R_1/7} \left(\begin{array}{cccc|c} 1 & 6/7 & 5/7 & 4/7 & 3/7 \\ 0 & 3 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

$$\xrightarrow{R_2/3} \left(\begin{array}{cccc|c} 1 & 6/7 & 5/7 & 4/7 & 3/7 \\ 0 & 1 & 4/3 & 5/3 & 2 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

$$\frac{5}{7} - \frac{4}{\cancel{3}} \cdot \frac{\cancel{6}\,2}{7}$$

$$\frac{5-8}{7} = -3/7$$

$$4/7 - \frac{5}{\cancel{3}} \cdot \frac{\cancel{6}\,2}{7}$$

$$-\frac{6}{7}$$

$$3/7 - 2 \cdot \frac{6}{7}$$

$$-\frac{9}{7}$$

$$\xrightarrow{R_1 - 6/7 R_2} \left(\begin{array}{cccc|c} 1 & 0 & -3/7 & -6/7 & -9/7 \\ 0 & 1 & 4/3 & 5/3 & 2 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

$$\xrightarrow{R_1 + 3/7 R_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4/3 & 5/3 & 2 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

$$-6/7 + 2 \cdot 3/7 = 0$$

$$-9/7 + 3 \cdot 3/7 = 0$$

$$\xrightarrow{R_2 - 4/3 R_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

$$5/3 - 4/3 \cdot 2 = -3/3 = -1$$

$$2 - 3 \cdot \frac{4}{3} = -2$$

RREF

7. Here is the augmented matrix of a linear system of equations. As usual, the variables are mentioned for your convenience.

$$\begin{pmatrix} x & y & z & w & \text{RHS} \\ \hline 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{pmatrix}$$

(a) Is this matrix in REF or RREF or neither of these?

Both

(b) Finish the solution process as needed and determine the complete solution of the system by filling in the answers below. If a variable is free, then enter the word "free" as its value. Be sure to show all calculations.

$x =$ $\boxed{x = 3 - 2y}$ $\quad x + 2y = 3 \quad x = 3 - 2y$

$y =$ $\boxed{\text{Free}}$

$z =$ $\boxed{4}$ $\quad z = 4 \quad 0x + 0y + 1z + 0w = 4$

$w =$ $\boxed{5}$ $\quad 0x + 0y + 0z + 1w = 5$

8. Your friend's calculator has never been the same since it went swimming last year. He asked it to solve Mr. Marjoram's problem, or at least bring the matrix to REF. It claims the answer is:

$$\left[\begin{array}{ccc|c} P & D & B & RHS \\ 3 & 4 & 5 & 180 \\ 0 & 1 & 11 & 180 \\ 0 & 0 & 1 & 15 \end{array}\right]$$

(a) Assume the calculator is right, and solve for P, D, and B.

$\boxed{(P = \underline{15}, D = \underline{15}, B = \underline{15})}$

$3P + 4D + 5B = 180 = 12 \cdot 15$

$B = 15$

$3P = 12 \cdot 15 - 9 \cdot 15$

$D + 11B = 180$

$3P = 3 \cdot 15$

$D = 180 - 11 \cdot 15 = 15$

$P = 15$

(b) Is your answer in part (a) actually right? Check your constraints in problem #5, and show that the production goals do or do not keep the machines busy.

$12 \cdot 15 + 16 \cdot 15 + 20 \cdot 15 = 48 \cdot 15 = 720$ ✓ $720 = 48 \cdot 15$

$\cancel{13 \cdot 15 + 17 \cdot}$ $13 + 17 + 18 = 48$

and $14 + 15 + 19 = 48$ so yes!

Your other friend's calculator hasn't been the same since it spent the night in the chemistry lab. It says the REF is:

$$\left[\begin{array}{ccc|c} P & D & B & RHS \\ 14 & 15 & 19 & 720 \\ 0 & 11 & 13 & 360 \\ 0 & 0 & 1 & 15 \end{array}\right] B$$

(c) Assume this calculator is also right, somehow. Solve for P, D, and B.

$\boxed{(P = \underline{15}, D = \underline{15}, B = \underline{15})}$

$11D + 13B = 360 = 24 \cdot 15$

$11D = (24 - 13)15 = 11 \cdot 15$ ✓

$\cancel{14 + 15 \cdot 1}$ $\cancel{48} \cdot$

$14P + 34 \cdot 15 = 48 \cdot 15$

we want $48 - 15 - 19 = 14$ so

$14P = 14 \cdot 15$