MA162: Finite mathematics

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February 8, 2012

Schedule:

- HW 2.4, 2.5 due Friday Feb 10, 2012
- HW 2.6, 3.1 due Friday Feb 17, 2012
- HW 3.2, 3.3 due Friday Feb 24, 2012
- HW 4.1 due Friday Mar 2, 2012
- Exam 2 is Monday, Mar 5, 2012 in CB106 and CB118

Today we will cover 2.5 and 2.6: matrix multiplication and division

2.5: Sizes for multiplication

- To multiply $A \cdot B$ we take the rows of A and multiply them against the columns of B
- We need each row of A to be the same length as each column of B
  They need to "match up"
- In other words, to multiply A and B, the number of columns of A must be equal to the number of rows of B
- 3×4 times 4×5 is good
  3×4 times 5×6 is not good, the rows of A have only 4 numbers, but the columns of B have 5
- If A is 3×4 and B is 4×5,
  then each little multiplication adds up 4 products

  (Rows $\times$ Columns)

2.5: Size for multiplication

- How big is $A \cdot B$?

- If A is 3×2 and B is 2×4 then $A \cdot B$ is 3×4:
  Each "little multiplication" adds up 2 products, and there are 3 rows of products, and 4 columns

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 7 + 2 \cdot 11 & 1 \cdot 8 + 2 \cdot 12 & 1 \cdot 9 + 2 \cdot 13 & 1 \cdot 10 + 2 \cdot 14 \\ 3 \cdot 7 + 4 \cdot 11 & 3 \cdot 8 + 4 \cdot 12 & 3 \cdot 9 + 4 \cdot 13 & 3 \cdot 10 + 4 \cdot 14 \\ 5 \cdot 7 + 6 \cdot 11 & 5 \cdot 8 + 6 \cdot 12 & 5 \cdot 9 + 6 \cdot 13 & 5 \cdot 10 + 6 \cdot 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 32 & 35 & 38 \\ 65 & 72 & 79 & 86 \\ 101 & 112 & 123 & 134 \end{bmatrix}$$

2.5: Easy word problem

- Two clients own some stocks:

$$\begin{array}{c} \\ \textit{Bill} \\ \textit{Jim} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{IBM} & \textit{Google} & \textit{Toyota} & \textit{Texaco} \end{array} \\ \begin{pmatrix} 18 & 16 & 12 & 14 \\ 12 & 18 & 11 & 12 \end{pmatrix} \end{array}$$

- The stocks have some prices today, yesterday, the day before

$$\begin{array}{c} \\ \textit{IBM} \\ \textit{Google} \\ \textit{Toyota} \\ \textit{Texaco} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Today} & \textit{Yesterday} & \textit{Daybefore} & \dots \end{array} \\ \begin{pmatrix} 3 & 3.01 & 2.99 & \dots \\ 4 & 3.99 & 3.99 & \dots \\ 5 & 5.01 & 5.01 & \dots \\ 1 & 1.02 & 1.03 & \dots \end{pmatrix} \end{array}$$

2.5: Easy word problem

- How much is each client's portfolio worth today?

$$\begin{array}{c} \\ Bill \\ Jim \end{array} \begin{array}{c} \begin{array}{cccc} IBM & Google & Toyota & Texaco \end{array} \\ \begin{pmatrix} 18 & 16 & 12 & 14 \\ 12 & 18 & 11 & 12 \end{pmatrix} \end{array} \bullet \begin{array}{c} \\ IBM \\ Google \\ Toyota \\ Texaco \end{array} \begin{array}{c} \begin{array}{cccc} Today & Yesterday & Daybefore & \ldots \end{array} \\ \begin{pmatrix} 3 & 3.01 & 2.99 & \ldots \\ 4 & 3.99 & 3.99 & \ldots \\ 5 & 5.01 & 5.01 & \ldots \\ 1 & 1.02 & 1.03 & \ldots \end{pmatrix} \end{array}$$

$$= \begin{array}{c} \\ Bill \\ Jim \end{array} \begin{array}{c} \begin{array}{cccc} Today & Yesterday & Daybefore & \ldots \end{array} \\ \begin{pmatrix} (18)(3)+(16)(4)+(12)(5)+(14)(1) & \ldots & \ldots & \ldots \\ (12)(3)+(18)(4)+(11)(5)+(12)(1) & \ldots & \ldots & \ldots \end{pmatrix} \end{array}$$

$$= \begin{array}{c} \\ Bill \\ Jim \end{array} \begin{array}{c} \begin{array}{cccc} Today & Yesterday & Daybefore & \ldots \end{array} \\ \begin{pmatrix} 192 & 192.42 & 192.20 & \ldots \\ 175 & 175.29 & 175.17 & \ldots \end{pmatrix} \end{array}$$

2.5: An easy multiplication, the identity

- There is a matrix that doesn't change things when it multiplies against them:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 11 & 12 & 13 & \ldots \\ 21 & 22 & 23 & \ldots \\ 31 & 32 & 33 & \ldots \\ 41 & 42 & 43 & \ldots \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 11 + 0 \cdot 21 + 0 \cdot 31 + 0 \cdot 41 & 12 & 13 & \ldots \\ 0 \cdot 11 + 1 \cdot 21 + 0 \cdot 31 + 0 \cdot 41 & 22 & 23 & \ldots \\ 0 \cdot 11 + 0 \cdot 21 + 1 \cdot 31 + 0 \cdot 41 & 32 & 33 & \ldots \\ 0 \cdot 11 + 0 \cdot 21 + 0 \cdot 31 + 1 \cdot 41 & 42 & 43 & \ldots \end{bmatrix}$$

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2.5: Multiplication to solve one equation

- $\frac{2}{3} \cdot \frac{3}{2} = 1$
- To solve $\frac{3}{2}x = 9$ we just multiply by $\frac{2}{3}$
- $x = \frac{2}{3}9 = 6$
- We are multiplying both sides by $\frac{2}{3}$
- Left side turns out nice and boring:

$$\frac{2}{3} \cdot (\frac{3}{2}x) = (\frac{2}{3} \cdot \frac{3}{2})x = (1)x = x$$

2.5: Multiplication to solve a system

- Matrix version:

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- To solve:

$$\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

- Just multiply:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} (1)(5) + (2)(7) \\ (1)(5) + (3)(7) \end{pmatrix} = \begin{pmatrix} 19 \\ 26 \end{pmatrix}$$

2.6: Matrix division

- There are several ways to do matrix division, see book for tricks
- We'll cover one systematic, basically easy way
- And we **already know it**, we just use RREF:
- If you know A and B, then to solve $AX = B$
  put the augmented matrix $(A|B)$ into RREF as $(I|X)$
- In other words, $RREF(A|B) = (I|X)$
- **inverses** are solving $AX = I$, $X = A^{-1}$, so we use RREF there too

2.6: Using RREF to solve the system

$$\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

- Make augmented matrix and RREF

$$\left(\begin{array}{cc|c} 3 & -2 & 5 \\ -1 & 1 & 7 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} -1 & 1 & 7 \\ 3 & -2 & 5 \end{array}\right) \xrightarrow{R_2 + 3R_1}$$

$$\left(\begin{array}{cc|c} -1 & 1 & 7 \\ 0 & 1 & 26 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{cc|c} -1 & 0 & -19 \\ 0 & 1 & 26 \end{array}\right) \xrightarrow{-R_1} \left(\begin{array}{cc|c} 1 & 0 & 19 \\ 0 & 1 & 26 \end{array}\right)$$

- Find inverse is almost exactly the same

$$\left(\begin{array}{cc|cc} 3 & -2 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|cc} -1 & 1 & 0 & 1 \\ 3 & -2 & 1 & 0 \end{array}\right) \xrightarrow{R_2 + 3R_1}$$

$$\left(\begin{array}{cc|cc} -1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{cc|cc} -1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow{-R_1} \left(\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right)$$

2.6: Why is the inverse useful?

- The inverse allows you to solve $AX = B$ using matrix multiplication instead of RREF
- $A^{-1}A = I$
- $A^{-1}AX = IX = X$
- If $AX = B$, then multiply both sides on the left by A^{-1}
  then $A^{-1}AX = A^{-1}B$
  so $X = A^{-1}B$
- Multiply by the inverse does the same thing as the long RREF
- Of course to find the inverse, we use RREF