

MA162: Finite mathematics

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SCHEDULE:

- HW 2.6, 3.1 due Friday Feb 17, 2012
- HW 3.2, 3.3 due Friday Feb 24, 2012
- HW 4.1 due Friday Mar 2, 2012
- Exam 2 is Monday, Mar 5, 2012 from 5pm to 7pm in CB106 and CB118
- *Most exams are up. Number grade on Blackboard and Mathclass.*
PDF on [yellow menu on mathclass](#).

Today we will cover 2.6: matrix inversion

Exam 2: Overview

- 22% Ch. 2, Matrix arithmetic
- 33% Ch. 3, Linear optimization with 2 variables
 - ① Graphing linear inequalities
 - ② Setting up linear programming problems
 - ③ Method of corners to find optimum values of linear objectives
- 45% Ch. 4, Linear optimization with millions of variables
 - ① Slack variables give us flexibility in RREF
 - ② Some RREFs are better (business decisions) than others
 - ③ Simplex algorithm to find the best one using row ops
 - ④ Accountants and entrepreneurs are two sides of the same coin

2.6: An obvious problem

- Mr. Marjoram is beginning to be interested in making some money
- He still wants to use up all the time on the machines
- But he also wants to know how much money he'll make in terms of the time on the machines
- Can we make him some sort of table?

	Panda	Dog	Bird		Sewing	Stuffing	Trimming
Revenue	\$2 per	\$3 per	\$4 per	Panda	12 min per	13 min per	14 min per
				Dog	16 min per	17 min per	15 min per
				Bird	20 min per	18 min per	19 min per

- How do we combine them to get a “available machine time” to “revenue” converter?
- “Every minute of sewing machine earns you so-and-so many dollars”

2.6: An obvious answer

- Row of revenue table matches column of timing table, so multiply?

$$\begin{array}{r} \text{Rev} \end{array} \begin{array}{ccc} P & D & B \\ (2 & 3 & 4) \end{array} \cdot \begin{array}{ccc} P & D & B \\ \begin{array}{ccc} \text{Sew} & \text{Stuff} & \text{Trim} \\ \left(\begin{array}{ccc} 12 & 13 & 14 \\ 16 & 17 & 15 \\ 20 & 18 & 19 \end{array} \right) \end{array} \end{array}$$

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- What does it mean? \$152 per minute of sewing machine time?
- If we have 12 hours, that is like $(\$152)(720) = \109440
That is pretty awesome!
And aren't there three machines? So like
 $(\$152 + \$149 + \$149)(720) = \324000

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And aren't there three machines? So like
 $(\$152 + \$149 + \$149)(720) = \324000
- He makes twenty-seven thousand dollars per hour!!!

2.6: An obvious answer is totally wrong

- We know from the practice exam that in fact he should make:
15 pandas, 15 dogs, and 15 birds
- He makes \$2 per Panda, \$3 per Dog, and \$4 per Bird so that is:

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- We know from the practice exam that in fact he should make: 15 pandas, 15 dogs, and 15 birds
- He makes \$2 per Panda, \$3 per Dog, and \$4 per Bird so that is:

$$\begin{array}{c} P \quad D \quad B \\ \text{Rev} \end{array} \begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \cdot \begin{array}{c} \text{Made} \\ P \\ D \\ B \end{array} \begin{pmatrix} 15 \\ 15 \\ 15 \end{pmatrix} = \begin{array}{c} \text{Made} \\ \text{Rev} \end{array} (30 + 45 + 60)$$

$$\begin{array}{c} \text{Made} \\ \text{Rev} \end{array} (135) \quad \text{just \$135 total}$$

2.6: So what is the answer?

- If we know how many animals he makes, we can find the total revenue
- Mr. Marjoram doesn't want to wait on us to solve for the number of animals everytime he needs to decide how many minutes of machine time to buy
- He needs a simple conversion. He pays you for this, and he is not interested in your excuses, so just go do it.

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- I think this is hard to do with pre-MA162 math, but it is just 4th grade math once you've learned your MA162

2.6: Use matrix arithmetic to help you think

- Call that 2,3,4 matrix R and that 12,13,14 matrix \hat{A} , and the available time is B .

$$R = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \quad \hat{A} = \begin{bmatrix} 12 & 13 & 14 \\ 16 & 17 & 15 \\ 20 & 18 & 19 \end{bmatrix} \quad B = \begin{bmatrix} 720 \\ 720 \\ 720 \end{bmatrix}$$

- We want some sort of converter $B \mapsto$ total revenue
- Our first guess was revenue = $R\hat{A}B$, but this was totally wrong

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$$X = \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix}$$

- Then the revenue is easy:

$$RX = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix} = [30 + 45 + 60] = [135]$$

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- Heck, now the final answer is easy: total revenue is $(RA^{-1})B$

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- Where is the 4th grade math answer you promised?

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- (a) Ask **the computer** what A^{-1} is:

$$A^{-1} = \frac{1}{144} \begin{bmatrix} -53 & 4 & 52 \\ -5 & 52 & -44 \\ 43 & -44 & 4 \end{bmatrix}$$

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- (b) multiply by R and by 60 minutes per hour

$$RA^{-1} = 60 \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \frac{1}{144} \begin{bmatrix} -53 & 4 & 52 \\ -5 & 52 & -44 \\ 43 & -44 & 4 \end{bmatrix} =$$
$$\frac{1}{4} \begin{bmatrix} 85 & -20 & -20 \end{bmatrix} \approx \begin{bmatrix} \$21.25 & \$5.00 & \$5.00 \end{bmatrix}$$

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- (c) Express it to Mr. Marjoram: Assuming there is a way to use up all the time on the machines, you earn \$21.25 per hour on the sewing machine, but you lose \$5 per hour each on the stuffing and trimming machines

2.6: Two questions

- How do we lose money by using the stuffing and trimming machines?
- Basically we have to make less profitable items to use up their time

Check that $(12)(21.25) - (12)(5) - (12)(5) = \135 is actually right

- Can we use a computer on the exam?

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- Can we use a computer on the exam?
- Nope. But you can use RREF!

$$(A|1) \xrightarrow{RREF} (1|A^{-1})$$

2.6: Using RREF to find an inverse

- Marjoram's A is a little messy, so let's work an easy one:

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- Set it up as an RREF problem:

$$(A|1) = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array} \right] \longrightarrow$$

Now RREF it!

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Now RREF it!

- $(A|1) = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array} \right] \xrightarrow{R_1 + 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & -1 & -3 & 1 \end{array} \right] \xrightarrow{-R_2} \left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array} \right] = (1|A^{-1})$