MA162: Finite mathematics

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SCHEDULE:

- HW 3.2, 3.3 due Friday Feb 24, 2012
- HW 4.1 due Friday Mar 2, 2012
- Exam 2 is Monday, Mar 5, 2012 from 5pm to 7pm in CB106 and CB118

Today we will cover 3.2: Linear programming problems

Exam 2: Overview

- 22% Ch. 2, Matrix arithmetic
- 33% Ch. 3, Linear optimization with 2 variables
    1. Graphing linear inequalities
    2. Setting up linear programming problems
    3. Method of corners to find optimum values of linear objectives
- 45% Ch. 4, Linear optimization with millions of variables
    1. Slack variables give us flexibility in RREF
    2. Some RREFs are better (business decisions) than others
    3. Simplex algorithm to find the best one using row ops
    4. Accountants and entrepreneurs are two sides of the same coin

3.2: Linear programming problems

- An LPP has three parts:
  - The variables (the business decision to be made)
  - The inequalities (the laws, constraints, rules, and regulations)
  - The objective (maximize profit, minimize cost)
- Setting up the problem will be **your job**!
- Reading the answer will be **your job**!
- The middle part is on the exam and you can do it!

3.2: Example 1. Production problem (1/2)

- Ace Novelty is a small company producing two products:
  - Monogrammed water bottles with custom cozy
  - Ornamental sphere and reptile pack (OSARP)
- It uses modern micro-manufacturing techniques including its:
  - MakerBot computer aided 3D printer
  - KnitBot-2010 computer controlled knitting machine
  - Assembly crew (people)

3.2: Example 1. Production problem (2/2)

- Each Water bottle realizes the company a profit of \$10
  Each OSARP realizes the company a profit of \$12

- Each item requires a certain amount of time (in minutes):

	3D Printer	KnitBot	Crew
Bottle	26	60	20
OSARP	62	30	40

- Time is short: Each day the company can only run the 3D printer 5 hours, the KnitBot 4 hours, and the crew 4 hours.

- The union is strong: The total machine time can only be three times as much as the human time

- How can you maximize profit without destroying the machines or ticking off the union?

3.2: Example 1. Setting it up (1/3)

- What do you actually have control over?
  Can you buy better machines?
  Can you bribe the union leader?
  Can you make time STAND STILL?!

- Maybe you should start by deciding how many bottles and how many OSARPs to make.

- The manager (you) sets the **Production Goals** in order to maximize profit legally

- We use **variables** to describe our decision:

  - X = the number of water bottles to make each day

  - Y = the number of OSARPs to make each day

3.2: Example 1. Setting it up (2/3)

- What constraints do we operate under?

$$\begin{aligned} 26X + 62Y &\leq 300 && \text{(3D printer time)} \\ 60X + 30Y &\leq 240 && \text{(KnitBot time)} \\ 20X + 40Y &\leq 240 && \text{(Human time)} \\ 26X - 28Y &\leq 0 && \text{(Union req.)} \end{aligned}$$

- Sanity: $X \geq 0$, $Y \geq 0$ (standard inequalities)
- Union requirement:
  Machine time is $26X + 60X + 62Y + 30Y = 86X + 92Y$ and
  Human time times three is $3(20X + 40Y) = 60X + 120Y$
  So requirement is $86X + 92Y \leq 60X + 120Y$, or

$$26X - 28Y \leq 0$$

3.2: Example 1. Setting it up (3/3)

- Ok, no problem. I have the answer. $X = 0$ and $Y = 0$. No rules are broken!
- We need a **goal**. We need an **objective**:
- **Maximize** the profit $P = 10X + 12Y$

- We can do a lot better than $X = 0$ and $Y = 0$ (with $P = 0$)
- Even $X = 1$ and $Y = 1$ is better! ($P = 22$ and no rules broken)

3.2: Example 1. Summary

- **Variables**:
  X = the number of water bottles to make each day
  Y = the number of OSARPs to make each day

- **Constraints**:

$$
\begin{aligned}
26X + 62Y &\le 300 && \text{(3D printer time)} \\
60X + 30Y &\le 240 && \text{(KnitBot time)} \\
20X + 40Y &\le 240 && \text{(Human time)} \\
26X - 28Y &\le 0 && \text{(Union req.)}
\end{aligned}
$$

  and $X \ge 0$, $Y \ge 0$

- **Objective**:
  Maximize the profit $P = 10X + 12Y$

- (Done! We just want to set the problem up!)

3.2: Example 2. Nutrition

- A Food-and-Nutrition-Science student was asked to design a diet for someone with iron and vitamin B deficiencies
- The student said the person should get at least 2400mg of iron, 2100mg of vitamin B_1, and 1500mg of vitamin B_2 (over 90 days)
- The student recommended two brands of vitamins:

	Brand A	Brand B	Min. Req
Iron	40mg	10mg	2400mg
B_1	10mg	15mg	2100mg
B_2	5mg	15mg	1500mg
Cost:	$0.06	$0.08	

- The client asked the student to recommend the **cheapest** solution
- How many pills of each brand should the person get in order to meet the nutritional requirements at the minimal cost?

3.2: Example 2. Setting it up

- **Variables**:
  X = number of pills of brand A
  Y = number of pills of brand B

- **Constraints**:

$$\begin{aligned} 40X + 10Y &\geq 2400 \qquad \text{(Iron)} \\ 10X + 15Y &\geq 2100 \qquad \text{(B1)} \\ 5X + 15Y &\geq 1500 \qquad \text{(B2)} \end{aligned}$$

  and $X \geq 0,\ Y \geq 0$

- **Objective**:
  Minimize cost $C = 0.06X + 0.08Y$

3.2: Example 3. Shipping costs

- You hit the big time, Mr. or Ms. Big Shot.
  You've got two manufacturing plants and two assembly plants
- Your assembly plants A1 and A2 need 80 and 70 engines
- Your production plants can produce up to 100 and 110 engines
- The shipping costs are:

	To assembly plant	
From	A1	A2
P1	100	60
P2	120	70

- How many engines should each production plant ship to each assembly plant to meet the production goals at the minimum shipping cost?

3.2: Example 3. Setting it up (1/3)

- What do you have control over? Four things?

$$\begin{aligned} X &= \text{Number of engines from P1 to A1} \\ Y &= \text{Number of engines from P1 to A2} \\ Z &= \text{Number of engines from P2 to A1} \\ \xi &= \text{Number of engines from P2 to A2} \end{aligned}$$

- But do we really need all these variables?
  How many engines does A1 even want?

- $X + Z = 80$ and $Y + \xi = 70$

- Why not just use X and Y?
  Z and ξ are just "the rest"

3.2: Example 3. Setting it up (2/3)

- What are the requirements?
- Sanity is complicated: $X \geq 0$, $Y \geq 0$, $Z \geq 0$, $\xi \geq 0$
- But wait, we got rid of Z and ξ!
  No big deal, just don't ship more than needed!
- Sanity: $0 \leq X \leq 80$ and $0 \leq Y \leq 70$
- Only other constraint is production capacity:
- $X + Y \leq 100$ from P1 capacity
- $Z + \xi \leq 110$ from P2 capacity
- Rewrite P2 as $(80 - X) + (70 - Y) \leq 110$ really just $40 \leq X + Y$

3.2: Example 3. Setting it up (3/3)

- What is the goal?
- Cost is complicated: $100X + 60Y + 120Z + 70\xi$
- Rewrite as $100X + 60Y + 120(80 - X) + 70(70 - Y)$
- Simplifies to $C = 9600 - 20X + 4900 - 10Y = 14500 - 20X - 10Y$
- Ok, but we need an executive summary, this was too long!

3.2: Example 3. Summary

- **Variables**:
  X = Number of engines from P1 to A1
  Y = Number of engines from P1 to A2
  $80 - X$ = Number of engines from P2 to A1 (the rest of A1's demand)
  $70 - Y$ = Number of engines from P2 to A2 (the rest of A2's demand)

- **Constraints**:

$$\begin{array}{rcll} X + Y & \leq & 100 & \text{(P1 max production)} \\ X + Y & \geq & 40 & \text{(P2 max production)} \\ X & \leq & 80 & \text{(sanity, A1 max demand)} \\ Y & \leq & 70 & \text{(sanity, A2 max demand)} \end{array}$$

  and $X \geq 0$, $Y \geq 0$

- **Objective**:
  minimize shipping cost $C = 14500 - 20X - 10Y$

3.2: Example 4. Fancy shipping

- Two plants P1 and P2 and three warehouses W1, W2, W3
- Shipping costs are in the following table:

	W1	W2	W3
P1	20	8	10
P2	12	22	18

- Maximum production and minimum requirements are:

	Prod.
P1	400
P2	600

	W1	W2	W3
Req	200	300	400

3.2: Example 4. Setting it up (1/3)

- We honestly have six variables! We'd run out of letters.
- $X_1, X_2, X_3, X_4, X_5, X_6$ are six different variables
- They are pronounced "Ecks One, Ecks Two, Ecks Three, . . ."
- The number is just like a serial number,
  it doesn't mean multiply or square or anything like that
- So our variables are:
  X_1 = number to ship from P1 to W1
  X_2 = number to ship from P1 to W2
  X_3 = number to ship from P1 to W3
  X_4 = number to ship from P2 to W1
  X_5 = number to ship from P2 to W2
  X_6 = number to ship from P2 to W3

3.2: Example 4. Setting it up (2 and 3/3)

- What are the constraints?
  Max production, and min reception

$$
\begin{aligned}
x_1 + x_2 + x_3 &\leq 400 && \text{(P1 max prod)} \\
x_4 + x_5 + x_6 &\leq 600 && \text{(P2 max prod)} \\
x_1 + x_4 &\geq 200 && \text{(W1 min supply)} \\
x_2 + x_5 &\geq 300 && \text{(W2 min supply)} \\
x_3 + x_6 &\geq 400 && \text{(W3 min supply)}
\end{aligned}
$$

  and $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_5 \geq 0$, and $x_6 \geq 0$.

- What is the objective?
  Minimize cost: $C = 20x_1 + 8x_2 + 10x_3 + 12x_4 + 22x_5 + 18x_6$