DEPARTMENT OF MATHEMATICS

Ma 162 Third Exam April 9, 2012

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	**Maximum Score**	**Actual Score**
1	12	
2	12	
3	12	
4	14	
5	12	
6	14	
7	12	
8	12	
Total	100	

NAME: _____ **Section:** _____

Last four digits of Student ID: _____

1. Simple Interest: $I = Prt$. Accumulation: $A = P(1 + rt)$.
2. Compound Interest Accumulation: $A = P(1 + i)^n$. Present value: $P = A(1 + i)^{(-n)}$.
3. Effective rate: $r_{\mathit{eff}} = (1 + \frac{r}{m})^m - 1$.
4. Annuity: Sum: $S = R\frac{((1+i)^n-1)}{i}$. Present value: $P = R\frac{(1-(1+i)^{-n})}{i}$.
5. Set counting: Two sets: $n(A \bigcup B) = n(A) + n(B) - n(A \bigcap B)$
   $n(A \bigcup B \bigcup C) = n(A) + n(B) + n(C) - n(A \bigcap B) - n(B \bigcap C) - n(C \bigcap A) + n(A \bigcap B \bigcap C)$.

(Practice version)

Make sure to show formulas. Answers with no work receive no credit.

1. Mr. Marjoram is temporarily short on money, but will have plenty in a week or two. His $100 electrical bill is due too soon, and he contemplates four options:

   (A) Pay it late, including a $5 late fee

   (B) Put it on his 36% APR credit card for one month (incurring 3% simple interest)

   (C) Get a loan from the pawn shop for 1% monthly interest and a $4 processing fee

   (D) Get a loan from Chek-N-Go at 432% APR for two weeks (incurring 16.80% simple interest)

   How much interest does each option incur? Which is the cheapest option?

   (A) Paying it late results in ☐ dollars above the original $100

   (B) Putting it on the credit card results in ☐ dollars above the $100

   (C) The loan from the pawn shop results in ☐ dollars abve the $100

   (D) The loan from Chek-N-Go results in ☐ dollars above the $100

   The cheapest option is: ☐

Make sure to show formulas. Answers with no work receive no credit.

2. Mrs. Oregano just received notification that her interest rate is changing from 6% APR to 24% APR, effective in three months. She expects to incur interest for the next six months. Assuming no further changes, how much interest will \$250.00 incur over the next six months: that is three months at 6% APR and three months at 24% APR, all compounded monthly.

   (a) After three months, the accumulated amount is ☐ dollars.

   (b) After six months, the accumulated amount is ☐ dollars.

   (c) The interest after six months is ☐ dollars.

   Mrs. Oregano has a limited time offer to transfer the present \$250.00 to an 15% APR account. How much interest would the \$250.00 incur after six months at 15% APR, compounded monthly?

   (d) The accumulated amount after six months would be ☐ dollars.

   (e) The interest would be ☐ dollars.

Make sure to show formulas. Answers with no work receive no credit.

3. Zach Crusoe is saving for the future. He has deposited \$0.20 per day into his 3.60% APR savings account (compounded daily, 360 days per year) for two years. How much is his account currently worth?

   (a) His account is worth $\boxed{}$ dollars.

   As he has gotten older, his responsibilities and allowance have increased. How much will his account be worth if he now deposits \$0.50 per day for the next year?

   (b) His account is worth $\boxed{}$ dollars after 3 years: 2 years of \$0.20 per day, and 1 year of \$0.50 per day

Make sure to show formulas. Answers with no work receive no credit.

4. Dr. Tarragon is buying his potatoes on credit and plans to purchase \$1000.00 worth of Yukon Golds at 12% APR compounded monthly. He needs to have them paid off by the end of the year, 7 months from now. How much is his monthly payment?

   (a) A monthly payment of $\boxed{}$ dollars will pay off the loan in 7 months.

   (b) Dr. Tarragon realizes he can only afford half that much. Rather than borrow half as much, he decides to pay it back half as quickly. A monthly payment of $\boxed{}$ dollars will pay off the loan in 14 months.

   (c) Dr. Tarragon finally admits he can only spare \$10 per month, which would only cover the interest. How much can he borrow and still pay it back in 5 years?

Make sure to show formulas. Answers with no work receive no credit.

5. Suppose that A, B and C are sets with 64, 57, and 58 members respectively. Calculate the indicated quantities. Display correct formulas or appropriate Venn diagrams.

   (a) If $A \bigcup B$ has 82 members, then $A \bigcap B$ has $\square$ members.

   (b) If it is further known that $A \bigcap C$ has 35 members, then $A \bigcup C$ has $\square$ members.

   (c) If, in addition, $B - C$ has 25 members, then $B \bigcap C$ has $\square$ members.

   (d) Finally, if we are given that the intersection of all three sets A, B, and C has 20 members, then the union of these three sets has $\square$ members.

Make sure to show formulas. Answers with no work receive no credit.

6. A survey of 100 College students were asked for their opinions about pizza. They were specifically whether they liked pepperoni, mushrooms, and garlic.

   - 43 students liked pepperoni.
   - 39 students liked mushrooms.
   - 40 students liked garlic.
   - 12 students liked both pepperoni and mushrooms.
   - 14 students liked both pepperoni and garlic.
   - 13 students liked both mushrooms and garlic.
   - 9 students liked all three toppings.

   Based on the above information, answer the following questions. You must show your calculations to receive credit.

   (a) How many students surveyed did not like any of the three toppings?

   Answer: [ ]

   (b) How many students surveyed liked at least two of the toppings?

   Answer: [ ]

Make sure to show formulas. Answers with no work receive no credit.

7. Your neighbor's poodle is putting on a fashion show for the neighborhood. She owns 4 collars, 10 ribbons, and 3 sweaters, and plans to model every combination for her audience.

   How many outfits does she plan to model in the show?

   Answer: [ ] outfits

   Before the show begins, you find an opportunity to shorten the show by hiding one article of clothing (lowering the number of combinations of outfits) but you only have time to hide one. To remove the highest number of possible outfits, should you hide a collar, a ribbon, or a sweater?

   Answer: [ ]

   How many outfits would be modeled now? **Answer:** [ ] outfits

Make sure to show formulas. Answers with no work receive no credit.

8. A fashion-conscious ant has 6 legs (3 pairs of feet) and 13 pairs of shoes. Assuming the ant always wears shoes in pairs (so front-left and front-right use the left and right shoe from a pair; similarly for the middle-left and middle-right; similarly for the back-left and back-right), how many ways can she get dressed in the morning?

   (a) **Answer:** [ ] outfits

   What if the ant is exploring more avant-garde fashion styles and decides the shoes do not need to match. Of course, she only wears left shoes on the left feet, and right shoes on the right feet, but doesn't care if the left shoe matches the right shoe. How many outfits are now available?

   (b) **Answer:** [ ] outfits

   What if the ant has decided matching shoes are for arachnids, and refuses to have any matching shoes, even on different feet! Of course left shoes are on the left feet, and right shoes are on the right feet. How many outfits are now available?

   (c) **Answer:** [ ] outfits

Answers:

#1 is a practical simple problem. Calculate the interst on several short term loans.

(A) results in $5 of interest. Bills are often very clear on this point.

(B) 3% of $100 is $3. Credit card statements often tally the finance charges, but of course, only after you've spent the money and made your decision.

(C) 1% of $100 is $1, but there is also a $4 fee, for a total of $5. Notice the fee produces most of the income for the pawn shop. This is apparently standard operating procedure, at least nationally.

(D) This option requires paying back the loan a little more quickly, but 16.80% of $100 is already $16.80, and so it is already much more expensive. This is a current rate from a "Checks Cashed" type of store.

Of all the options, (B) the credit card is cheapest, resulting in only a $3 penalty for not having any money. Cheaper still would be to have the money currently, but without proper precautions (that apparently were not taken) it can be hard to have the money when you need it.

Answers:

#2 Each month the debt rises by the monthly interest rate. This is calculated by multiplying by $1 + i$, where i is the monthly interest rate. The first three months are at $i = 6\%/12 = 0.5\% = 0.005$, and the next three are at $i = 24\%/12 = 2\% = 0.02$. The alternative has six months at $i = 15\%/12 = 0.0125$

(a) $P = \$250.00$, $n = 3$, $i = 0.005$, $A = P(1 + i)^n = 250(1.005)^3 = 253.7687812 = \$253.77$

(b) $P = \$253.77$, $n = 3$, $i = 0.02$, $A = P(1 + i)^n = 253.77(1.02)^3 = \$269.30$; more simply this is just $\$250.00(1.005)^3(1.02)^3 = \$269.30$

(c) The interest is the extra money, not the original \$250, so that is $\$269.30 - \$250.00 = \$19.30$

(d) $P = \$250.00$, $n = 6$, $i = 0.0125$, $A = P(1 + i)^n = 250.00(1.0125)^6 = \$269.35$

(e) The interest is just the extra \$19.35

So the 6% first, followed by 24% results in less interest than the average 15% over the same period (though not much less).

Answers:

#3 In this problem, not only does old money get interest, but each day there is more new money added. In other words we have a seady cash flow, and need annuity formulas.

(a) $R = 0.20$, $i = 3.60\%/360 = 0.0001$ per day, $n = 720$ days, $S = R((1+i)^n - 1)/(i) = 0.20(1.0001^7 20 - 1)/(0.0001) = \$149.30$

(b) This requires splitting his money into two accounts (and there is more than one way to do this). One method is to cease payments on his 20-cent annuity, and just let that accumuated money sit for a year, $\$149.30(1.0001)^3 60 = \$154.77$, and then run a 50-cent annuity for a year $\$0.50(1.0001^7 20 - 1)/(0.0001) = \$183.27$, for a total of \$338.04.

Alternatively in (b) you could run the 20-cent for 3 years, and an additional 30-cent for 1 year.

Answers:

#4 This time the big money is before the small payments, so we need the present value of a future cash flow.

1. $P = \$1000.00$, $i = 12\%/12 = 0.01$ per month, $n = 7$ months, $R = P/((1 - (1 + i)^(- n))/(i)) = \$1000.00/((1 - 1.01^{-7})/(0.01)) = \$148.63$ per month.
2. $n = 14$, $R = \$1000.00/((1 - 1.01^{-14})/(0.01)) = \$76.90$
3. $n = 60$, $R = \$10$, $P = R(1 - (1 + i)^{(-n)})/(i) = 10(1 - 1.01^{-60})/(0.01) = \$449.55$

Notice how taking twice as long did cut the payment in half: half of \$148.63 is about \$72, so there was an extra 3 and a half dollars per month for taking twice as long.

Answers:

#5 This is an inclusion-exclusion problem from the homework.

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, so $82 = 64 + 57 - n(A \cap B)$, so $n(A \cap B) = 64 + 57 - 82 = 39$; if we add up A and B we should get the union, except that the intersection is double counted. To fix it, we just subtract the intersection. 82 is not actually equal to $64 + 57$, it is 39 too small. Hence we must have counted 39 people twice.

2. $n(A \cup C) = n(A) + n(C) - n(A \cap C) = 64 + 58 - 35 = 87$

3. $n(B) = n(B \cap C) + n(B - C)$, so $57 = n(B \cap C) + 25$, so $n(B \cap C) = 57 - 25 = 32$; everyone in B is either in C or not. 57 in B, 25 not in C, so the other 32 are in C.

4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) = 64 + 57 + 58 - 39 - 35 - 32 + 20 = 93$

Answers:

#6 This is another inclusion-exclusion problem. It is most easily solved but just figuring out what is going on first. Draw the Venn-diagram and fill it out, starting at the center.

9 like all three, 13 like mushrooms and garlic, so apparently 4 of those 13 like mushrooms and garlic, but not pepperoni. So the center of the Venn diagram has a 9, and the small petals have 4, 5, 3. Of the 40 who like garlic, 9 like all three, 4 like only mushroom too, and 5 like only pepperoni too, leaving $40 - 9 - 4 - 5 = 22$ who only like mushrooms. The outer petals have 22, 26, and 23.

(a) 100 students, the petals total to 9+4+5+3+22+26+23 = 92, so that leaves 8 who didn't like any topping.

(b) The inner petals total to 9+4+5+3 = 21

Answers:

#7 This is a multiplication principle problem.

(a) Every collar can be matched with every ribbon, giving $(4)(10) = 40$ possibilities for the head and neck, each of which can be matched with any of the sweaters, for a total of $(4)(10)(3) = 120$ outfits.

(b) One might as well calculate all three: $(4-1)(10)(3) = 90$, $(4)(10-1)(3) = 108$, $(4)(10)(3-1) = 80$, so removing a sweater gives the least number of outfits remaining.

Make sure to label your calculations clearly. Numerical answers with no justication receive NO credit.

Answers:

#8 This is a permutation-combination problem.

1. 13 pairs of shoes, 3 pairs of feet, how many ways can we arrange them? $(13)(12)(11) = 1716$

2. Well, we do part (a) for the left feet and for the right feet. Every one of the 1716 possibilities for the left feet comes with 1716 possibilities for the right feet, so $(13)(12)(11)(13)(12)(11) = (1716)(1716) = 2944656$

3. Once we choose the left feet, we have removed three pairs of shoes (even though we just throw away the right shoes), so we have $(10)(9)(8)$ possibilities using the remaining 10 pairs of shoes: $(13)(12)(11)(10)(9)(8) = 1235520$.

Make sure to label your calculations clearly. Numerical answers with no justication receive NO credit.