MA162: Finite mathematics

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April 4, 2012

Schedule:

- HW 6.2,6.3 are due Fri, April 6th, 2012
- Exam 3 is Monday, Apr 9th, 5:00pm-7:00pm in CB106 and CB118.

Today we will cover 6.4: Permutations

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - **Permutations**

6.4: Trifecta!

- Some people bet on horse races, a “Trifecta” bet is common
- You predict the first, second, and third place winners, in order.
- There are 14 contenders: **A**ccounting We Will Go, **B**usiness Planner, **C**orporate Finance, **D**ebt Sealing, **E**conomy Model, **F**iscal Filly, **G**ross Domestic Pony, **H**orse Resources, **I**nitial Pony Offering, **J**ust Another Horsey, **K**arpay Deeum, **L**OL Street, **M**arkety Mark, and **N**o Chance Vance
- Which ones will you choose? **A, B, C** or **L, N, E**?
- How many possibilities?

6.4: Counting the possibilities

______	______	______
1st	2nd	3rd

- There are three places

6.4: Counting the possibilities

14		
1st	2nd	3rd

- There are three places
- There are 14 possibilities for first place,

6.4: Counting the possibilities

14	13	
1st	2nd	3rd

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place

6.4: Counting the possibilities

14	13	12
1st	2nd	3rd

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place
- and only 12 left for third place

6.4: Counting the possibilities

$$\underset{\text{1ST}}{\underline{\quad 14 \quad}} \quad \underset{\text{2ND}}{\underline{\quad 13 \quad}} \quad \underset{\text{3RD}}{\underline{\quad 12 \quad}} = 2184$$

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- That is $(14)(13)(12) = 2184$ total possibilities

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- but only 13 left for second place
- and only 12 left for third place
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- If you bet 1000 times, only a 1 in 3 chance of winning at least once

6.4: Club officers

- The Variety Club has a President, a Vice President, a Secretary, and a Treasurer
- The V.C. has 6 members: Art, Ben, Cin, Dan, Eve, and Fin.
- But every day they want to assign a different set of officers
- Can they make it a year without exactly repeating the officer assignments?
- So maybe ABCD, then ABCE, then ABCF, then ABDC, then . . .

6.4: Counting the assignments

$\overline{\quad\quad\quad}$ $\overline{\quad\quad\quad}$ $\overline{\quad\quad\quad}$ $\overline{\quad\quad\quad}$

Pres *Vice* *Sec.* *Trs.*

- There are four positions, and order matters

6.4: Counting the assignments

6			
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- There are 6 people available to president each day

6.4: Counting the assignments

6	5		
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- There are 6 people available to president each day
- There are 5 people left to be VP

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6	5	4	
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- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary

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$$\underset{Pres}{\underline{\quad 6 \quad}} \quad \underset{Vice}{\underline{\quad 5 \quad}} \quad \underset{Sec.}{\underline{\quad 4 \quad}} \quad \underset{Trs.}{\underline{\quad 3 \quad}} = 360$$

- There are four positions, and order matters
- There are 6 people available to president each day
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- There are $(6)(5)(4)(3) = 360$ possible assignments

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- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer
- There are $(6)(5)(4)(3) = 360$ possible assignments
- Not enough for a calendar year, but certainly for a school year!

6.4: Always down by one?

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  How many ways can a P and a VP be chosen
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- There are two positions:

$$\underset{Pres}{\underline{\hspace{2cm}}} \quad \underset{Trs.}{\underline{\hspace{2cm}}}$$

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- There are ten people eligible for president

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- There are ten people eligible for president
- But only five people left for vice president

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$$\underset{\textit{Pres}}{\underline{\quad 10 \quad}} \quad \underset{\textit{Trs.}}{\underline{\quad 5 \quad}} = 50$$

- There are ten people eligible for president
- But only five people left for vice president
- That is $(5)(10) = 50$ different officer assignments

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- Well, there are ten shoes trying out for the first part, but whomever you choose also eliminates their stunt double
- So eight for the second part, and six for the third; $10*8*6 = 480$ ways.

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- But you don't care what order they are in. So that is four ways:

$$\{L1, L2\}, \{L1, R2\}, \{R1, L2\}, \{R1, R2\}$$

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- 4*2 ways counting order, then divide by two to ignore order

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- How many ways can one rearrange the letters of KENTUCKY?
- Well, a little different since there are two Ks
- 8! ways if we keep track of which K is which, then divide by two since each word like KENTUCKY appears twice as kENTUCKY and KENTUCkY.

$$8!/2 = 20160$$

6.4: Team players

- If there are 15 able bodied players, and we need to choose 11 of them to be on the field. We want four forwards, three midfielders, three defenders, and one goalie. We let the players themselves dynamically decide on the left/right/center. How many selections are possible?

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- (15)(14)(13)(12) choices of forwards counting order, but (4)(3)(2)(1) ways of re-ordering them, so $(15)(14)(13)(12)/((4)(3)(2)(1)) = 15!/(11!4!) = 1365$ ways ignoring order

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- Then 5 ways of choosing the goalie.

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- $(11)(10)(9)$ choices of midfielders with $(3)(2)(1)$ ways to reorder, so $(11)(10)(9)/((3)(2)(1)) = 11!/(8!3!) = 165$ ways ignoring order
- Then $8!/(5!3!) = 56$ ways of choosing defenders ignoring order
- Then 5 ways of choosing the goalie.
- Total is: $(1365)(165)(56)(5)$ ways of choosing the first string

6.4: Summary

- We learned to handle symmetries in our counting, especially **permutations**, and **combinations**.
- Make sure to complete the homework ASAP, and begin work on the practice exam
- Be ready to discuss the practice exam next class; bring a copy