MA162: Finite mathematics

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University of Kentucky

April 18, 2012

Schedule:

- HW 7A, 7B due Fri, April 20, 2012
- HW 7C due Fri, April 27, 2012
- Final exam, Wed May 2, 2012 from 8:30pm to 10:30pm

Today we will cover 7.3: Rules of probability

Final Exam Breakdown

- Chapter 7: Probability
  - Counting based probability
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  - Empirical probability
  - Conditional probability
- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP

7.2: Just count for probability

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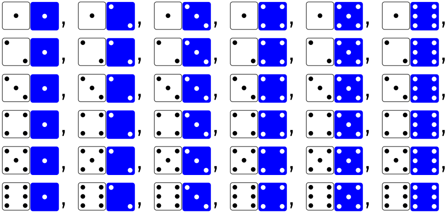

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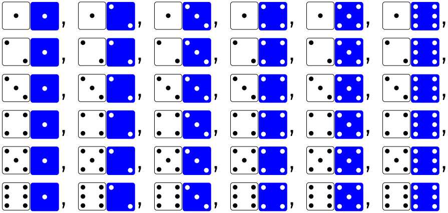

- The second row and the fifth column work: $P = \frac{6+6-1}{(6)(6)} = \frac{11}{36}$

7.2: Crazy counting

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$$P(\text{at least } 2) = \frac{C(4,2)C(20,1) + C(4,3)}{C(24,3)} = \frac{30}{506} + \frac{1}{506} = \frac{31}{506}$$

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- $P(E - F) = P(E) - P(E \cap F) = 40\% - 10\% = 30\%$

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- $Pr(E) = Pr(E \cap F) + Pr(E - F)$
- Every counting problem formula you can imagine has a probability counterpart

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$$\frac{91}{216} = 1 - \left(1 - \frac{1}{6}\right)^3$$

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