DEPARTMENT OF MATHEMATICS

Ma 162 Second Exam October 18, 2010

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure that your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations. You must show your work to receive credit.
Put your answers in the answer boxes provided, and show your work.
If your answer is not in the box or if you have no work to support your answer, you will receive no credit.
The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	18	
2	16	
3	16	
4	16	
5	18	
6	16	
Total	100	

Please fill in the information below.

NAME: ______________________________ **Section:** ____________
Last four digits of Student ID: __________________

1. Consider the following matrices and answer the questions.

   In each case, either calculate the expression or explain why it is not defined.

$$P = \begin{bmatrix} -1 & -3 & -4 \\ 4 & -2 & -3 \\ 5 & -1 & 3 \end{bmatrix} \quad Q = \begin{bmatrix} 4 & 0 & 1 \\ -4 & 2 & -5 \end{bmatrix} \quad L = \begin{bmatrix} 4 & -4 \\ 1 & -3 \\ -4 & 5 \end{bmatrix} \quad M = \begin{bmatrix} 5 & -1 \\ -3 & 4 \end{bmatrix}$$

(a) $M^2 - 9\,M$

Answer:

(b) LP

Answer:

(c) QP

Answer:

(d) MQ

Answer:

(e) $11\,Q - 5\,L$

Answer:

2. (a) Clearly state the used formula or show your work.

Find the inverse of the matrix $A = \begin{bmatrix} 1 & -4 \\ -4 & -4 \end{bmatrix}$.

Answer:

(b) Use your calculator or usual algorithm to find the inverse of the matrix $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$.

Answer:

(c) Suppose that $A^{-1} = \begin{bmatrix} -1 & -2 & -1 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$

and $B = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix}$.

Determine the solution X of the equation $AX = B$.

Answer:

3. Set this problem up, by stating the chosen variables, the function to be maximized and **all** the inequalities. **Do not solve the problem.**

The "Officeware" company has three lines of computer desks called A,B,C.

Each desk of type A requires 1.7 hours of fabrication 0.8 hours of assembly work and 1.6 hours of finishing.

Each desk of type B requires 1.3 hours of fabrication 0.8 hours of assembly work and 1.4 hours of finishing.

Each desk of type C requires 1.4 hours of fabrication 1.0 hours of assembly work and 2.0 hours of finishing.

The company has 150 hours available for fabrication, 250 hours for assembly and 300 hours for finishing in its shop.

If the profits per desk for the three lines A,B,C are $61, 68, 47$ dollars respectively, how many desks of each type should be produced?

Define each variable here:

Maximize: Profit $P =$

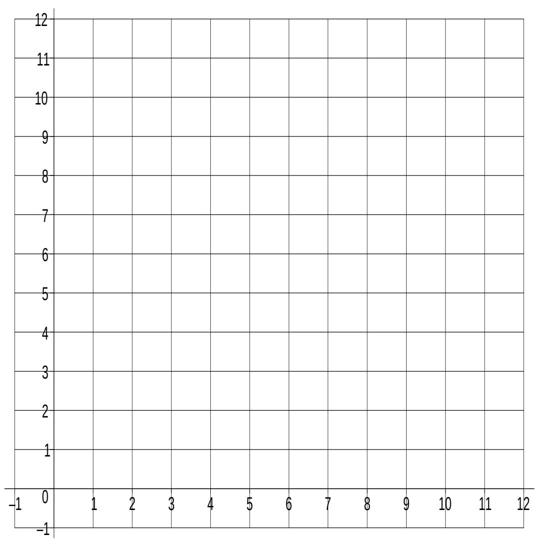
Subject to:

4. i) Sketch and shade the region described by the inequalities. Compute the coordinates of the corner points and mark them on your graph.

$$0 \le x \ , \ 0 \le y$$

$$9 \le x + y$$

$$x + 1.5\, y \le 11.5$$



ii) Find the maximum value of the function, $P = x + 3.0\, y$ on the region.

Answer: $P =$ ☐ at $x =$ ☐, $y =$ ☐.

5. Here is an intermediate tableau associated with a maximal Linear Programming Problem(LPP).

x	y	z	s	t	P	constants
0	1	2/3	1	$-1/6$	0	4
1	6	2	0	1	0	60
0	-2	3	0	1	1	60

i) Circle the pivot element and carry out the next iteration of the simplex method.

ii) Using your answer in the first part, report the solution to the original maximal LPP.

Value of P= $\boxed{}$ (x , y , z)=($\boxed{}$, $\boxed{}$, $\boxed{}$)

6. You are given the minimization problem:

Minimize the objective function: $C = 5\,x + 6\,y + 10\,z$

Subject to:

$$10 \le 2\,x + y + 5\,z$$

$$6 \le 4\,x + y + z$$

$x \ge 0, y \ge 0$ and $z \ge 0$

The final tableau **for the dual problem** is:

u	v	x	y	z	P	constants
0	1	$\frac{5}{18}$	0	$-1/9$	0	$\frac{5}{18}$
0	0	$-2/9$	1	$-1/9$	0	$\frac{34}{9}$
1	0	$-1/18$	0	$2/9$	0	$\frac{35}{18}$
0	0	$\frac{10}{9}$	0	$\frac{14}{9}$	1	$\frac{190}{9}$

Using this give the solution to the primal problem (i.e. original minimal LPP):

Value of C= $\boxed{}$

The point: (x , y , z)=($\boxed{}$, $\boxed{}$, $\boxed{}$)

1 Answer Key for exam2v-3

1. $\diamond$ (a) $\begin{bmatrix} -17 & 0 \\ 0 & -17 \end{bmatrix}$ (b) DNE (c) $\begin{bmatrix} 1 & -13 & -13 \\ -13 & 13 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 24 & -2 & 10 \\ -28 & 8 & -23 \end{bmatrix}$ (e) DNE

2. $\diamond$ (a) $\begin{bmatrix} 1/5 & -1/5 \\ -1/5 & -1/20 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 8 & -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 8 \\ 0 \\ -6 \end{bmatrix}$

3. $\diamond$ $$\begin{array}{c} P = 61\,x + 68\,y + 47\,z \\ 0 \le x \ ,\ 0 \le y \\ \frac{17}{10}\,x + \frac{13}{10}\,y + 7/5\,z \le 150 \\ 4/5\,x + 4/5\,y + z \le 250 \\ 8/5\,x + 7/5\,y + 2\,z \le 300 \end{array}$$

4. $\diamond$ $P = 19.0$ at $x = 4$ $y = 5$. $[[9,0],[11.5,0],[4,5]]$

5. $\diamond$ (i) Pivot in [1,2] position $\begin{array}{ccccccc} 0 & 1 & 2/3 & 1 & -1/6 & 0 & 4 \\ 1 & 0 & -2 & -6 & 2 & 0 & 36 \\ 0 & 0 & 13/3 & 2 & 2/3 & 1 & 68 \end{array}$ (ii) P= 68 (x,y,z) $[36,4,0]$

6. $\diamond$ P= $\frac{190}{9}$ (x,y,z) $[\frac{10}{9}, 0, \frac{14}{9}]$