1. Producing 15 items costs \$300, but producing 20 items costs \$320. Assuming a linear model of production costs, how much would producing 16 items cost?

20 − 15 = 5 more items
320 − 300 = \$20 more dollars

one more item costs \$20/5 = \$4 more

16 items is \$304

2. Where do the lines given by the following equations intersect? $x+y = 12$ and $2x+3y = 31$

$$\begin{matrix} x+y=12 \\ 2x+3y=31 \end{matrix} \xrightarrow{2R_1} \begin{matrix} 2x+2y=24 \\ 2x+3y=31 \end{matrix} \xrightarrow{R_2-R_1} \begin{matrix} 2x+2y=24 \\ 0+y=7 \end{matrix} \xrightarrow{R_1-2R_2} \begin{matrix} 2x+0=10 \\ 0+y=7 \end{matrix}$$

$$\xrightarrow{\frac{1}{2}R_1} \begin{matrix} x+0=5 \\ 0+y=7 \end{matrix} \longrightarrow (x=5, y=7)$$

3. Matrix arithmetic. Do the following calculations if possible. If impossible, explain why.

(a) Add $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$ Undefined Sizes must be equal

2×3 $\quad 3 \times 1$

(b) Multiply $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 1(10)+2(20)+3(30) \\ 4(10)+5(20)+6(30) \end{bmatrix} = \begin{bmatrix} 140 \\ 320 \end{bmatrix}$

$2 \times 3 \quad 3 \times 1 = 2 \times 1$

(c) Add $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 70 & 80 & 90 \\ 100 & 110 & 120 \end{bmatrix} = \begin{bmatrix} 71 & 82 & 93 \\ 104 & 115 & 126 \end{bmatrix}$

$2 \times 3 \quad 2 \times 3 = 2 \times 3$

(d) Multiply $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 70 & 80 & 90 \\ 100 & 110 & 120 \end{bmatrix}$ Undefined. Inner sizes must match.

$2 \times 3 \neq 2 \times 3$

4. Write the following equations in matrix form:

$$x + y = z$$
$$2x = y + z$$
$$3x + 4y + 5z = 30$$

$$X + y - z = 0$$
$$2x - y - z = 0$$
$$3x + 4y + 5z = 30$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & RHS \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ 3 & 4 & 5 & 30 \end{array}\right] \end{array}$$

5. A system of equations is represented by the matrix

$$\left[\begin{array}{ccc|c} x & y & z & RHS \\ \hline 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

(a) Write out the system of equations

$$1x + 0y + 0z = 3$$
$$0x + 1y + 0z = 4$$
$$0x + 0y + 1z = 5$$

$\longrightarrow$ $x = 3$, $y = 4$, $z = 5$

(b) Solve it: $x =$ 3, $y =$ 4, $z =$ 5

6. A system of equations is represented by the matrix

$$\left[\begin{array}{ccc|c} x & y & z & RHS \\ \hline 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

(a) Write out the system of equations

$$x + y = 3$$
$$z = 5$$

$\longrightarrow$

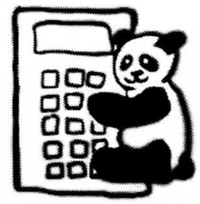
$x = 3 - y$

$y =$ FREE

$z = 5$

(b) Solve it: $x =$ $3 - y$, $y =$ FREE, $z =$ 5

7. Panda-money-em specializes in production of panda bear themed financial calculators. The fixed costs of production total to \$1000, while the marginal costs are only \$10 per calculator. If the calculators sell for \$50 per calculator (and boy do they sell!), what is the break-even production and the break-even cost? Make sure to show your work clearly! Exhibit the cost function and the revenue function, and describe how they are related to solving this problem.



$C(x) = 10x + 1000$

$R(x) = 50x$

$P(x) = 40x - 1000$

Need to make $40x = 1000$

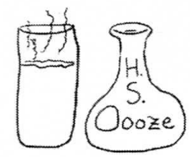
$\boxed{x = 25}$ calculators to break-even

Costs $10(25) + 1000 = \boxed{1250}$

Earns $50(25) = 1250$

Net $= 0$, break-even

8. Hill Street Oooze specializes in the production of food like products from various chemical substances. It has 4 main ingredients: Red, Green, White, and Pulsing oozes. It has 3 main products: Mutant Mango, Neon Nectarine, and OMG Orange. Additionally it has two main dispenser machines: the Front Machine and the Side Machine. Describe how much of each ingredient is used at each of the machines given the following ingredients list and sales records. Show your work. It is a good idea to write this out in terms of matrices.



	Ingredients per order				Sales record		
	Red	Green	White	Pulsing	Front Machine	Side Machine	
Mutant Mango	6 oz	0 oz	1 oz	1 oz	100 orders	60 orders	Mutant Mango
Neon Nectarine	4 oz	2 oz	0 oz	2 oz	200 orders	50 orders	Neon Nectarine
OMG Orange	3 oz	1 oz	1 oz	3 oz	300 orders	40 orders	OMG Orange

$$\begin{array}{c|c} & \begin{array}{ccc} M & N & O \end{array} \\ \begin{array}{c} Red \\ G \\ W \\ P \end{array} & \begin{bmatrix} 6 & 4 & 3 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \end{array} \times \begin{array}{c|c} & \begin{array}{cc} F & S \end{array} \\ \begin{array}{c} M \\ N \\ O \end{array} & \begin{bmatrix} 100 & 60 \\ 200 & 50 \\ 300 & 40 \end{bmatrix} \end{array} = \begin{array}{c|c} & \begin{array}{cc} \text{Front} & S \end{array} \\ \begin{array}{c} R \\ G \\ W \\ P \end{array} & \begin{bmatrix} 6(100)+4(200)+3(300) & \\ 0(100)+2(200)+1(300) & \\ & \\ & \end{bmatrix} \end{array}$$

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	Front	Side
Red	2300 oz	680 oz
Green	700 oz	140 oz
White	400 oz	100 oz
Pulsing	1400 oz	280 oz

9. The data analysts have done a best-linear-model-fit to the data on the suppliers and found that supply X is currently governed by $X = 45P + 100$ as long as the price P remains between \$5 and \$10 per unit. The demand is handled by another department, and they appear to be on vacation. You know that at \$5 per unit, 500 will be demanded, and at \$10 per unit only 100 will be demanded.

(a) What is the demand equation if one uses a linear model for demand?

Marginal Demand $= \dfrac{500-100}{5-10} = \dfrac{400}{-5} = -80$

$X = 500 - 80(P-5)$

start at 500

lose 80 every dollar above 5

expanded: $X = 900 - 80P$

(b) What is the equillibium price and equillibrium demand?

$45P + 100 = 900 - 80P$

$125P = 800$

$P = 800/125 = \$6.40$

(supply) $X = 45(\$6.40) + 100 = 388$ "units"

(ez demand) $X = 500 - 80(1.40) = 388$

(expanded) $X = 900 - 80(6.40) = 388$

So $X=388$ units supplied and demanded at $P = \$6.40$, the equillibrium

10. Before recent adjustments to accounting practices, departments were required to spend all the money on each budget line each fiscal year. At the end of one year, the department discovered it had about \$1000 in each of three budget lines: Labor, Materials, and Storage. It had three ongoing projects: the Old project, the New project, and the previously Forgotten project. Each project spends a certain amount of each budget per day, and the department needs to tell each project how many days of work it needs to do before the year is up.

	Resource usage			
	Old	New	Forgotten	Budget
Labor	\$9	\$18	\$27	\$1035
Materials	\$8	\$17	\$28	\$1005
Storage	\$9	\$18	\$28	\$1050

(a) Describe the variables for the problem. If a variable is "x" make sure your description explains clearly what "$x = 4$" means.

X = O = number of days to do Old project
N = " " " " " New "
F = " " " " " Forgotten "

(b) Write down the equations for the problem in terms of your variables.

$$9X + 18N + 27F = 1035$$
$$8X + 17N + 28F = 1005$$
$$9X + 18N + 28F = 1050$$

(c) Solve the equations (preferably using matrices)

$$\left[\begin{array}{ccc|c} 9 & 18 & 27 & 1035 \\ 8 & 17 & 28 & 1005 \\ 9 & 18 & 28 & 1050 \end{array}\right] \xrightarrow[\text{wimpy}]{\frac{1}{9}R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 115 \\ 8 & 17 & 28 & 1005 \\ 9 & 18 & 28 & 1050 \end{array}\right] \xrightarrow[R_3-9R_1]{R_2-8R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 115 \\ 0 & 1 & 4 & 85 \\ 0 & 0 & 1 & 15 \end{array}\right]$$

$$\xrightarrow[R_2-4R_3]{R_1-3R_3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 70 \\ 0 & 1 & 0 & 25 \\ 0 & 0 & 1 & 15 \end{array}\right] \xrightarrow{R_1-2R_2} \begin{array}{c} \begin{array}{cccc} X & N & F & \text{RHS} \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 25 \\ 0 & 0 & 1 & 15 \end{array}\right] \end{array}$$

$$X = 20, \quad N = 25, \quad F = 15$$

(d) What are the department's orders for the projects in plain English?

Do 20 days of work on the old project,
25 days of work on the new project,
and 15 days of work on the forgotten project.