MA162: Finite mathematics

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SCHEDULE:

- HW 3.2, 3.3 are due Friday, Oct 5th, 2012
- HW 4.1, 4.2 are due Friday, Oct 12th, 2012
- Exam 2 is Monday, Oct 15th, 5:00pm-7:00pm in BS107 and BS116.
- Exam grades on blackboard, PDFs on mathclass.

Today we will cover 3.2: setting up word problems

3.2: Linear programming problems

- An LPP has three parts:
  - The variables (the business decision to be made)
  - The inequalities (the laws, constraints, rules, and regulations)
  - The objective (maximize profit, minimize cost)
- Setting up the problem will be **your job**!
- Reading the answer will be **your job**!
- The middle part is on the exam and you can do it!

3.2: Example 1. Production problem (1/2)

- Ace Novelty is a small company producing two products:
    - Monogrammed water bottles with custom cozy
    - Ornamental sphere and reptile pack (OSARP)
- It uses modern micro-manufacturing techniques including its:
    - MakerBot computer aided 3D printer
    - KnitBot-2010 computer controlled knitting machine
    - Assembly crew (people)

3.2: Example 1. Production problem (2/2)

- Each Water bottle realizes the company a profit of \$10
  Each OSARP realizes the company a profit of \$12

- Each item requires a certain amount of time (in minutes):

	3D Printer	KnitBot	Crew
Bottle	26	60	20
OSARP	62	30	40

- Time is short: Each day the company can only run the 3D printer 5 hours, the KnitBot 4 hours, and the crew 4 hours.

- The union is strong: The total machine time can only be three times as much as the human time

- How can you maximize profit without destroying the machines or ticking off the union?

3.2: Example 1. Setting it up (1/3)

- What do you actually have control over?
  Can you buy better machines?
  Can you bribe the union leader?
  Can you make time STAND STILL?!

- Maybe you should start by deciding how many bottles and how many OSARPs to make.

- The manager (you) sets the **Production Goals** in order to maximize profit legally

- We use **variables** to describe our decision:

  - X = the number of water bottles to make each day

  - Y = the number of OSARPs to make each day

3.2: Example 1. Setting it up (2/3)

- What constraints do we operate under?

$$\begin{aligned} 26X + 62Y &\leq 300 && \text{(3D printer time)} \\ 60X + 30Y &\leq 240 && \text{(KnitBot time)} \\ 20X + 40Y &\leq 240 && \text{(Human time)} \\ 26X - 28Y &\leq \quad 0 && \text{(Union req.)} \end{aligned}$$

- Sanity: $X \geq 0$, $Y \geq 0$ (standard inequalities)

- Union requirement:
  Machine time is $26X + 60X + 62Y + 30Y = 86X + 92Y$ and
  Human time times three is $3(20X + 40Y) = 60X + 120Y$
  So requirement is $86X + 92Y \leq 60X + 120Y$, or

$$26X - 28Y \leq 0$$

3.2: Example 1. Setting it up (3/3)

- Ok, no problem. I have the answer. $X = 0$ and $Y = 0$. No rules are broken!
- We need a **goal**. We need an **objective**:
- **Maximize** the profit $P = 10X + 12Y$

- We can do a lot better than $X = 0$ and $Y = 0$ (with $P = 0$)
- Even $X = 1$ and $Y = 1$ is better! ($P = 22$ and no rules broken)

3.2: Example 1. Summary

- **Variables**:
  X = the number of water bottles to make each day
  Y = the number of OSARPs to make each day

- **Constraints**:

$$\begin{aligned} 26X + 62Y &\le 300 && \text{(3D printer time)} \\ 60X + 30Y &\le 240 && \text{(KnitBot time)} \\ 20X + 40Y &\le 240 && \text{(Human time)} \\ 26X - 28Y &\le 0 && \text{(Union req.)} \end{aligned}$$

  and $X \ge 0$, $Y \ge 0$

- **Objective**:
  Maximize the profit $P = 10X + 12Y$

- (Done! We just want to set the problem up!)

3.2: Example 2. Nutrition

- A Food-and-Nutrition-Science student was asked to design a diet for someone with iron and vitamin B deficiencies
- The student said the person should get at least 2400mg of iron, 2100mg of vitamin B_1, and 1500mg of vitamin B_2 (over 90 days)
- The student recommended two brands of vitamins:

	Brand A	Brand B	Min. Req
Iron	40mg	10mg	2400mg
B_1	10mg	15mg	2100mg
B_2	5mg	15mg	1500mg
Cost:	\$0.06	\$0.08	

- The client asked the student to recommend the **cheapest** solution
- How many pills of each brand should the person get in order to meet the nutritional requirements at the minimal cost?

3.2: Example 2. Setting it up

- **Variables**:
  X = number of pills of brand A
  Y = number of pills of brand B

- **Constraints**:

$$\begin{aligned} 40X + 10Y &\geq 2400 \qquad &\text{(Iron)} \\ 10X + 15Y &\geq 2100 \qquad &\text{(B1)} \\ 5X + 15Y &\geq 1500 \qquad &\text{(B2)} \end{aligned}$$

  and $X \geq 0,\ Y \geq 0$

- **Objective**:
  Minimize cost $C = 0.06X + 0.08Y$

3.2: Example 3. Shipping costs

- You hit the big time, Mr. or Ms. Big Shot.
  You've got two manufacturing plants and two assembly plants
- Your assembly plants A1 and A2 need 80 and 70 engines
- Your production plants can produce up to 100 and 110 engines
- The shipping costs are:

	To assembly plant	
From	A1	A2
P1	100	60
P2	120	70

- How many engines should each production plant ship to each assembly plant to meet the production goals at the minimum shipping cost?

3.2: Example 3. Setting it up (1/3)

- What do you have control over? Four things?

$$\begin{aligned} X &= \text{Number of engines from P1 to A1} \\ Y &= \text{Number of engines from P1 to A2} \\ Z &= \text{Number of engines from P2 to A1} \\ \xi &= \text{Number of engines from P2 to A2} \end{aligned}$$

- But do we really need all these variables?
  How many engines does A1 even want?

- $X + Z = 80$ and $Y + \xi = 70$

- Why not just use X and Y?
  Z and ξ are just "the rest"

3.2: Example 3. Setting it up (2/3)

- What are the requirements?
- Sanity is complicated: $X \geq 0$, $Y \geq 0$, $Z \geq 0$, $\xi \geq 0$
- But wait, we got rid of Z and ξ!
  No big deal, just don't ship more than needed!
- Sanity: $0 \leq X \leq 80$ and $0 \leq Y \leq 70$
- Only other constraint is production capacity:
- $X + Y \leq 100$ from P1 capacity
- $Z + \xi \leq 110$ from P2 capacity
- Rewrite P2 as $(80 - X) + (70 - Y) \leq 110$ really just $40 \leq X + Y$

3.2: Example 3. Setting it up (3/3)

- What is the goal?
- Cost is complicated: $100X + 60Y + 120Z + 70\xi$
- Rewrite as $100X + 60Y + 120(80 - X) + 70(70 - Y)$
- Simplifies to $C = 9600 - 20X + 4900 - 10Y = 14500 - 20X - 10Y$
- Ok, but we need an executive summary, this was too long!

3.2: Example 3. Summary

- **Variables**:
  X = Number of engines from P1 to A1
  Y = Number of engines from P1 to A2
  $80 - X$ = Number of engines from P2 to A1 (the rest of A1's demand)
  $70 - Y$ = Number of engines from P2 to A2 (the rest of A2's demand)

- **Constraints**:

$$\begin{array}{rcrl} X + Y & \leq & 100 & \quad \text{(P1 max production)} \\ X + Y & \geq & 40 & \quad \text{(P2 max production)} \\ X & \leq & 80 & \quad \text{(sanity, A1 max demand)} \\ Y & \leq & 70 & \quad \text{(sanity, A2 max demand)} \end{array}$$

  and $X \geq 0$, $Y \geq 0$

- **Objective**:
  minimize shipping cost $C = 14500 - 20X - 10Y$

3.2: Example 4. Fancy shipping

- Two plants P1 and P2 and three warehouses W1, W2, W3
- Shipping costs are in the following table:

	W1	W2	W3
P1	20	8	10
P2	12	22	18

- Maximum production and minimum requirements are:

	Prod.
P1	400
P2	600

	W1	W2	W3
Req	200	300	400

3.2: Example 4. Setting it up (1/3)

- We honestly have six variables! We'd run out of letters.
- $X_1, X_2, X_3, X_4, X_5, X_6$ are six different variables
- They are pronounced "Ecks One, Ecks Two, Ecks Three, . . ."
- The number is just like a serial number,
  it doesn't mean multiply or square or anything like that
- So our variables are:
  X_1 = number to ship from P1 to W1
  X_2 = number to ship from P1 to W2
  X_3 = number to ship from P1 to W3
  X_4 = number to ship from P2 to W1
  X_5 = number to ship from P2 to W2
  X_6 = number to ship from P2 to W3

3.2: Example 4. Setting it up (2 and 3/3)

- What are the constraints?
  Max production, and min reception

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 400 & &\text{(P1 max prod)} \\ x_4 + x_5 + x_6 &\leq 600 & &\text{(P2 max prod)} \\ x_1 + x_4 &\geq 200 & &\text{(W1 min supply)} \\ x_2 + x_5 &\geq 300 & &\text{(W2 min supply)} \\ x_3 + x_6 &\geq 400 & &\text{(W3 min supply)} \end{aligned}$$

  and $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_5 \geq 0$, and $x_6 \geq 0$.

- What is the objective?
  Minimize cost: $C = 20x_1 + 8x_2 + 10x_3 + 12x_4 + 22x_5 + 18x_6$