# DEPARTMENT OF MATHEMATICS

Ma 162 Second Exam October 15, 2012 (practice)

**Instructions:** No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

| Problem | Maximum Score | Actual Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 5 | |
| 6 | 5 | |
| 7 | 15 | |
| 8 | 15 | |
| 9 | 20 | |
| 10 | 20 | |
| Total | 100 | |

**NAME:** JACK **Section:** 999

**Last four digits of Student ID:** 9999

1. Is $(x = 2, y = 4)$ on the correct side of $10x + 30y \leq 100$? Explain why or why not.

$10(\overset{x}{2}) + 30(\overset{y}{4}) = 20 + 120 = 140 \not\leq 100$

so $(x=2, y=4)$ is on the wrong side of $10x+30y \leq 100$.

Just Test the point in the inequality.

2. Is $(x = 2, y = 4)$ a feasible solution to "maximize $P = 1.10x+1.20y$ subject to $3x+y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$"? Is it optimal? Explain why or why not.

Check Each one:

$3(2) + 4 = 10 \leq 15$ ✓

$5(2) + 5(4) = 30 \leq 35$ ✓

$2 \geq 0$ ✓

$4 \geq 0$ ✓

So it is feasible.

$(P = 1.10(2) + 1.20(4) = \$7)$

It is not optimal since it is "interior", it is not on any edge.

We can increase $y$ to 5 and stay feasible: $(x=2, y=5)$

$3(2) + 5 = 11 \leq 15$ ✓

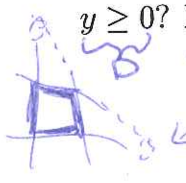
$5(2) + 5(5) = 35 \leq 35$ ✓

$2 \geq 0$ ✓

$5 \geq 0$ ✓

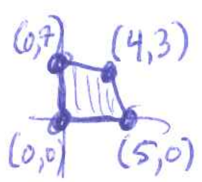
but $P = 1.10(2) + 1.20(5) = \$8.20$ is better!

3. What are the corners of the feasible region described by $\underbrace{3x + y \leq 15}_{A}$, $\underbrace{5x + 5y \leq 35}_{B}$, $\underbrace{x \geq 0}_{C}$, $y \geq 0$? Make sure to show at least one full calculation.



Should be 4 corners, two wrong ones

AB: $\begin{cases} 3x+y = 15 \\ 5x+5y = 35 \end{cases} \rightarrow \begin{matrix} 3x+y=15 \\ x+y=7 \end{matrix} \rightarrow \begin{matrix} 2x+0 = 8 \\ x+y=7 \end{matrix} \rightarrow \begin{matrix} x=4 \\ y=3 \end{matrix}$




AC: $\begin{matrix} 3x+y=15 \\ x=0 \end{matrix} \rightarrow \begin{matrix} y=15 \\ x=0 \end{matrix}$ but $(0,15)$ breaks B, so not corner

AD: $\begin{matrix} 3x+y=15 \\ y=0 \end{matrix} \rightarrow \begin{matrix} x=5 \\ y=0 \end{matrix}$ $(5,0)$ is feasible, so corner.

BC: $\begin{matrix} 5x+5y=35 \\ x=0 \end{matrix} \rightarrow \begin{matrix} y=7 \\ x=0 \end{matrix}$ $(0,7)$ is feasible, so corner

BD:

CD:

Problems 4,5,6 refer to this simplex tableau:

$$\left(\begin{array}{ccc|c cccc|c|c} x & y & z & a & b & c & d & P & RHS \\ \hline 1 & 1 & 1 & 0 & 2 & 0 & 0 & 0 & 11 \\ 2 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 12 \\ 3 & 0 & 1 & 0 & 4 & 0 & 1 & 0 & 13 \\ 4 & 0 & 1 & 1 & 5 & 0 & 0 & 0 & 14 \\ \hline -5 & 0 & 1 & 0 & -6 & 0 & 0 & 1 & 15 \end{array}\right)$$

4. What is the basic solution indicated by this simplex tableau? Explain why it is feasible, but not optimal.

$x=0$ (Free), $y=11$, $z=0$ (free), $a=14$, $b=0$ (free), $c=12$, $d=13$, $P=15$

It is feasible because all variables are $\geq 0$.
Equivalently, the RightHandSide is all $\geq 0$

It is not optimal because $P = 15 \underline{+5x} - z \underline{+6b}$, neither $x=0$, nor $b=0$ is optimal.

5. Which columns in this simplex tableau are eligible for pivoting? What happens if you pivot on a wrong column?

The X and B columns are eligible, because they are still profitable (neg in bottom row means $P = 15 \underline{+5x} \cdots$, so $x=0$ is not optimal).

If we pivot on a wrong column, we lower the profit!
The ratio from #6 times the botton row # is how the profit changes, because of the row op!

6. Assuming we pivot the first column, which rows are eligible for pivoting? What happens if you pivot on a wrong row?

The ratios are

| | |
|---|---|
| 11/1 | 11 |
| 12/2 | 6 |
| 13/3 | 4.3... |
| 14/4 | 3.5 |

and the smallest, non-negative ratio is the 4th row.

Only the 4th (no ties)

If we choose the wrong row, the RHS will get negative #s so the basic solution won't be feasible.

7. Do the row ops to pivot on the 1st column, 1st row, even if this is not the right row or column.

(It is an ok column, wrong row)

$$
\left(\begin{array}{ccc|cccc|c|c}
\mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} & \mathbf{P} & \mathbf{RHS} \\
\hline
1 & 1 & 1 & 0 & 2 & 0 & 0 & 0 & 11 \\
2 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 12 \\
3 & 0 & 1 & 0 & 4 & 0 & 1 & 0 & 13 \\
4 & 0 & 1 & 1 & 5 & 0 & 0 & 0 & 14 \\
\hline
-5 & 0 & 1 & 0 & -6 & 0 & 0 & 1 & 15
\end{array}\right)
$$

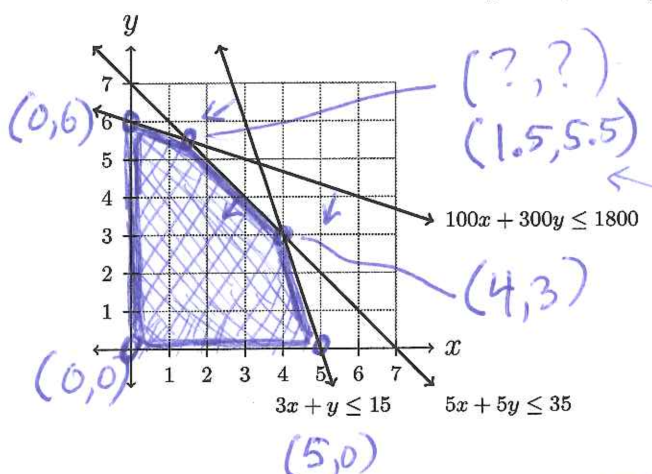
$R_1$ is fine
$R_2 - 2R_1$
$R_3 - 3R_1$
$R_4 - 4R_1$
$R_5 + 5R_1$

$$
\left(\begin{array}{cccccccc|c}
1 & 1 & 1 & 0 & 2 & 0 & 0 & 0 & 11 \\
0 & -2 & -1 & 0 & -1 & 1 & 0 & 0 & -10 \\
0 & -3 & -2 & 0 & -2 & 0 & 1 & 0 & -20 \\
0 & -4 & -3 & 1 & -3 & 0 & 0 & 0 & -30 \\
\hline
0 & 5 & 6 & 0 & 4 & 0 & 0 & 1 & 70
\end{array}\right)
$$

← Oops! Not Feasible. Every row with a lower Ratio is broken!

This one is "optimal" but not feasible. Make a lot of money by breaking the law. Wrong Row!

8. Maximize $P = 1.10x + 1.20y$ subject to $100x + 300y \leq 1800$, $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$. Make sure to (1) shade the region, (2) label the corners, (3) label where the maximum occurs and how big it is, and (4) why it must be the maximum.




$\begin{cases} 5x + 5y = 35 \\ 100x + 300y = 1800 \end{cases} \rightarrow \begin{array}{l} x + y = 7 \\ x + 3y = 18 \end{array} \rightarrow \begin{array}{l} x + y = 7 \\ 2y = 11 \end{array}$

$\rightarrow \begin{cases} x = 1.5 \\ y = 5.5 \end{cases}$

| X | Y | P |
|---|---|---|
| 0 | 0 | 0.00 |
| 5 | 0 | 5.50 |
| 4 | 3 | 8.00 |
| 1.5 | 5.5 | 8.25 |
| 0 | 6 | 7.20 |

The maximum must occur at a corner, so just check the corners. The best corner is (1.5, 5.5) with P = \$8.25, the maximum.

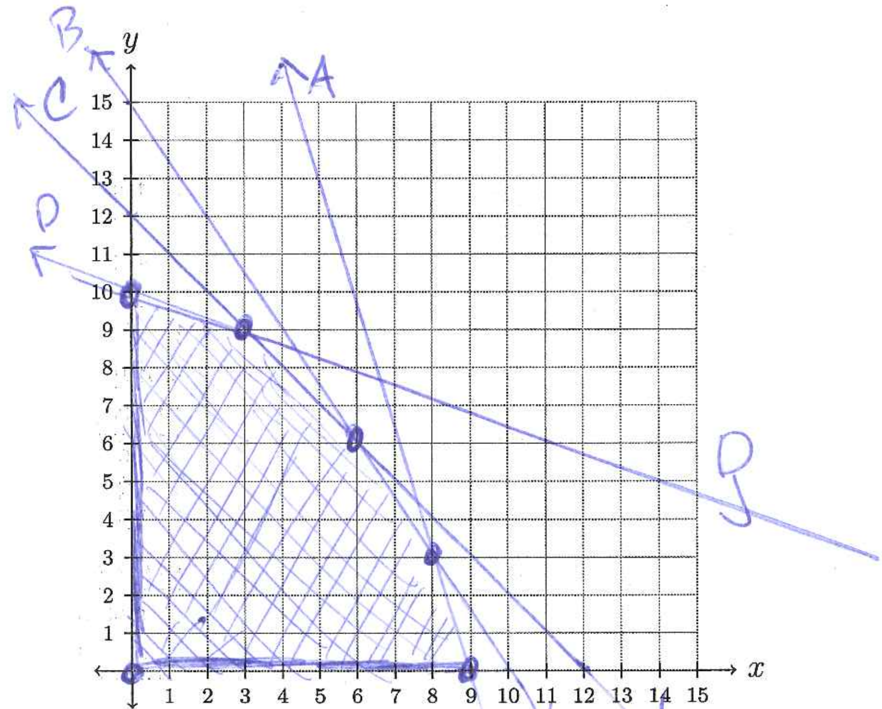
Clearly show your work. Not only find the right answer, but convince me you know what you are doing.

9. Vincent is trying to maximize profit using his limited resources. He has four limited resources: 27 tubes of Amarillo paint, 30 tubes of Berry red paint, 12 Canvases, and 30 tubes of Dark blue paint. He only knows how to paint two things: Sunshine and Lollipops, but he can sell as many as he can paint, earning a profit of \$10 per Sunshine painting and \$12 per Lollipops painting. Each painting requires a few tubes of paint:

| | Amarillo | Berry Red | Canvases | Dark Blue | Profit |
|---|---|---|---|---|---|
| Sunshine | 3 | 3 | 1 | 1 | 10 |
| Lollipops | 1 | 2 | 1 | 3 | 12 |
| Inventory | 27 | 30 | 12 | 30 | |

**Give a recommendation to maximize his profit using only his limited resources:**

| | | | |
|---|---|---|---|
| Number of Sunshine paintings: | 3 | Bottom line Profit: | \$138 |
| Number of Lollipops paintings: | 9 | | |
| Leftover tubes of Amarillo paint: | 9 | | |
| Leftover tubes of Berry red paint: | 3 | | |
| Leftover Canvases: 12-12 | 0 | | |
| Leftover tubes of Dark blue paint: ~~3+27~~ 30-30 | 0 | | |




Convert Table To Inequalities

Sun Lolli Inv

A: $3x + y \le 27$ — (9,0), (8,3)

B: $3x + 2y \le 30$ — (10,0), (0,15)

C: $x + y \le 12$ — (0,12), (12,0)

D: $x + 3y \le 30$ — (0,10), (3,9)

A B C

Corners are

| x | y | P |
|---|---|---|
| 0 | 10 | 0(10) + 10(12) = 120 |
| 3 | 9 | 3(10) + 9(12) = 138 |
| 6 | 6 | 6(10) + 6(12) = 132 |
| 8 | 3 | 8(10) + 3(12) = 116 |
| 9 | 0 | 9(10) + 0(12) = 90 |
| 0 | 0 | 0(10) + 0(12) = 0 |

So make x=3 Sun, y=9 Lol for \$138.

Use 3(3) + 9(1) Amarillo, so 27-18=9 left

10. Vincent is trying to maximize profit using his limited resources and his brilliant new painting. He has four limited resources: 27 tubes of Amarillo paint, 30 tubes of Berry red paint, 12 Canvases, and 30 tubes of Dark blue paint. He now knows how to paint **three** things: Sunshine, Lollipops, and Rainbows Everywhere, and he can sell as many as he can paint, earning a profit of \$10 per Sunshine painting, \$12 per Lollipops painting, and an impressive \$20 per Rainbows Everywhere painting. Each painting requires a few tubes of paint:

| | Amarillo | Berry Red | Canvases | Dark Blue | Profit |
|---|---|---|---|---|---|
| Sunshine | 3 | 3 | 1 | 1 | 10 |
| Lollipops | 1 | 2 | 1 | 3 | 12 |
| Rainbows | 2 | 2 | 1 | 2 | 20 |
| Inventory | 27 | 30 | 12 | 30 | |

**Hint:** When choosing a pivot column, in this case try the book's advice of choosing the most (immediately) profitable column first.

Which paintings should Vincent produce to maximize his profit using his limited resources?

$$\begin{array}{c} \\ (A) \\ (B) \\ (C) \\ (D) \\ (P) \end{array} \begin{array}{ccc|cccc|c|c} S & L & R & A & B & C & D & P & RHS \\ 3 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 27 \\ 3 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 30 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 12 \\ 1 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 30 \\ -10 & -12 & -20 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}$$

S, L, R are profitable. Choose R.

$27/2 = 13.5$
$30/2 = 15$
$12/1 = 12$ ← lowest ratio
$30/2 = 15$

$R_3$ is fine

$$\begin{array}{l} \\ R_1 - 2R_3 \\ R_2 - 2R_3 \\ R_3 \checkmark \\ R_4 - 2R_3 \\ R_5 + 20R_3 \end{array} \begin{array}{ccc|cccc|c|c} S & L & R & A & B & C & D & P & RHS \\ 1 & -1 & 0 & 1 & 0 & -2 & 0 & 0 & 3 \\ 1 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & 6 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 12 \\ -1 & 1 & 0 & 0 & 0 & -2 & 1 & 0 & 6 \\ 10 & 8 & 0 & 0 & 0 & 20 & 0 & 1 & 240 \end{array}$$

All $\geq 0$ so done!

Basic Solution: $S=0, L=0, R=12,$
$A=3, B=6, C=0, D=6$
$P=240$

$P = 240 - 10S - 8L - 20C$

Every Sunshine costs us \$10 profit,
every lollipop costs us \$8 profit,
every unused canvas costs us \$20.

So Vincent should make 12 Rainbow Paintings, and 0 sunshine, 0 Lolipops.
This leaves 3 Tubes of A, 6 tubes of B, 6 tubes of D leftover,
but uses all the canvasses.