MA162: Finite mathematics

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Schedule:

- Exam 1 is Today, Oct 15th, 5:00pm-7:00pm in
  BS107 (Tuesday REC) and BS116 (Thursday REC).
- Alternate exam (appt. only) Monday, Oct 15th, 3:00pm-5:00pm in CB212.

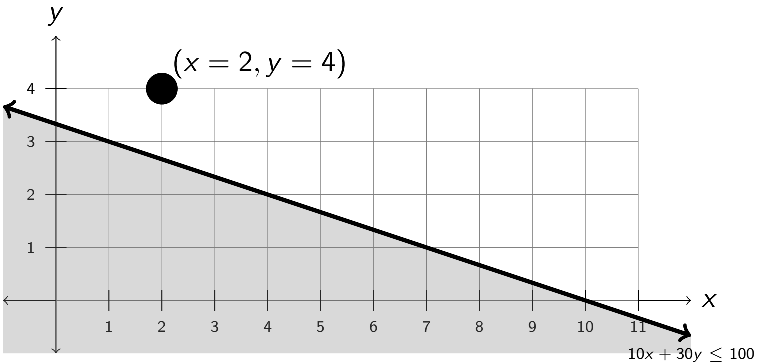
Today we will review the practice exam.

Practice exam: #1

- Is $(x = 2, y = 4)$ on the correct side of $10x + 30y \leq 100$? Explain why or why not.

Practice exam: #1

- Is $(x = 2, y = 4)$ on the correct side of $10x + 30y \leq 100$? Explain why or why not.
- Just check: $10(2) + 30(4) = 20 + 120 > 100$, so no.



Practice exam #2

- Is $(x = 2, y = 4)$ a feasible solution to "maximize $P = 1.10x + 1.20y$ subject to $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$"? Is it optimal? Explain why or why not.

Practice exam #2

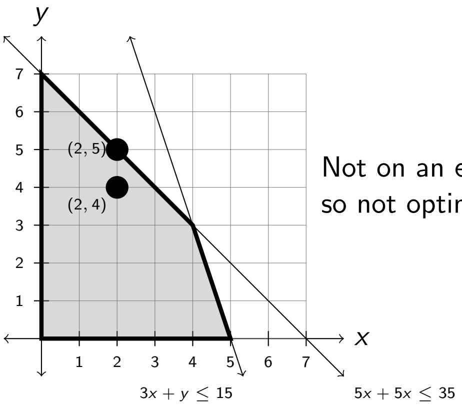
- Is $(x = 2, y = 4)$ a feasible solution to "maximize $P = 1.10x + 1.20y$ subject to $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$"? Is it optimal? Explain why or why not.
- Just check:

$$3(2) + 4 = 10 < 15, \text{ good}$$
$$5(2) + 5(4) = 30 < 35, \text{ good}$$
$$2 > 0,\ 4 > 0, \text{ good, good.}$$

- So yes, feasible, but in fact it was very very feasible, so it cannot be optimal!
- $P(2, 4) = 1.10(2) + 1.20(4) = \$7.00$, but $(2, 5)$ is also feasible $(11 \leq 15,\ 35 \leq 35,\ 2 \geq 0,\ 5 \geq 0)$ and $P(2, 5) = 1.10(2) + 1.20(5) = \$8.20$.
- Very feasible meant room for improvement!

Practice exam #2: picture version

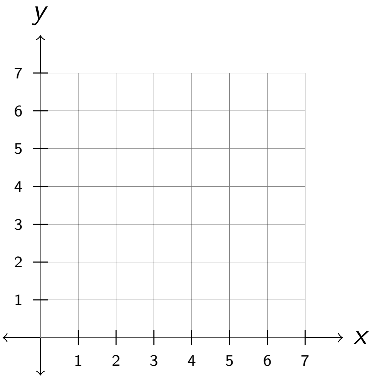
- Is $(x = 2, y = 4)$ a feasible solution to "maximize $P = 1.10x + 1.20y$ subject to $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$"? Is it optimal? Explain why or why not.



Not on an edge, so not optimal.

Practice exam #3

- What are the corners of the feasible region described by $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$? Make sure to show at least one full calculation.



Practice exam #3

- What are the corners of the feasible region described by $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$? Make sure to show at least one full calculation.

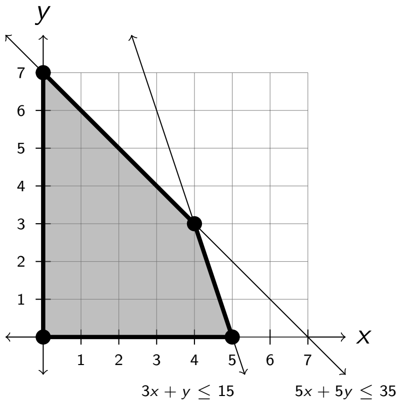
Each corner is the intersection of a pair of lines. Some intersections are not corners, but you can tell this by checking if they are feasible.

A good corner:

$$\left\{ \begin{array}{r} 3x + \ \ y = 15 \\ 5x + 5y = 35 \end{array} \right\} \xrightarrow{R_2/5} \left\{ \begin{array}{r} 3x + y = 15 \\ x + y = 7 \end{array} \right\}$$

$$\xrightarrow{R_1 - R_2} \left\{ \begin{array}{rr} 2x & = 8 \\ x + y & = 7 \end{array} \right\} \xrightarrow[R_2 - R_1]{R_1/2} \left\{ \begin{array}{rr} x & = 4 \\ y & = 3 \end{array} \right\}$$

Now check that it is feasible. Well two of the inequalities are definitely equalities; that is how we found the point. The other two are just $x \geq 0$ and $y \geq 0$, and sure enough $3 \geq 0$ and $4 \geq 0$, so this is a feasible intersection, a **corner**.



Practice exam #4

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
−5	0	1	0	−6	0	0	1	15

- What is the basic solution indicated by this simplex tableau?

- Explain why it is feasible, but not optimal.

Practice exam #4

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
−5	0	1	0	−6	0	0	1	15

- What is the basic solution indicated by this simplex tableau?

  The basic solution is the one where all free variables are set to 0.

  The general solution is
  $(x = \text{FREE}, y = 11 - x - z - 2b, z = \text{FREE}, a = 14 - 4x - z - 5b, b = \text{FREE}, c = 12 - 2x - z - 3b, d = 13 - 3x - z - 4b, P = 15 + 5x - z + 6b)$

  The basic solution is
  $(x = 0, y = 11, z = 0, a = 14, b = 0, c = 12, d = 13, P = 15)$

- Explain why it is feasible, but not optimal.

  It is feasible since all the x, y, z, a, b, c, d numbers are non-negative. It is not optimal since $P = 15 + 5x + \ldots$ but we set $x = 0$!

Practice exam #5

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
-5	0	1	0	-6	0	0	1	15

- Which columns in this simplex tableau are eligible for pivoting?

- What happens if you pivot on a wrong column?

Practice exam #5

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
-5	0	1	0	-6	0	0	1	15

- Which columns in this simplex tableau are eligible for pivoting?

  Since $P = 15 + 5X - Z + 6B$ both X and B are good to pivot on.

- What happens if you pivot on a wrong column?

  If we pivot on Z, then the profit will go down (assuming we stay feasible). All of the other variables are already "pivotted": they only have a single 1 in their column, so pivotting them has no effect on anything at all.

Practice exam #6

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
-5	0	1	0	-6	0	0	1	15

- Assuming we pivot the first column, which rows are eligible for pivoting?

- What happens if you pivot on a wrong row?

Practice exam #6

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
−5	0	1	0	−6	0	0	1	15

- Assuming we pivot the first column, which rows are eligible for pivoting?
  We no longer want to assume $X = 0$. How big can X be? We calculate the ratios: $11/1 = 11, 12/2 = 6, 13/3 = 4.3, 14/4 = 3.5, 15/(-5) = \text{neg.}$ The smallest non-negative ratio was 3.5 from the fourth row, so that is the only row we can pivot on. (Sometimes there are ties. Also $0/5 = 0$ would be smallest, but $5/0 = \infty$ is either very big or negative, so never the right answer.)
- What happens if you pivot on a wrong row?
  We make X too big, and violate a constraint if we choose a ratio that is bigger than the smallest non-negative one; the particular constraint will be the one in the row with a smaller non-negative ratio. If we choose a negative ratio it can make X negative. If we choose an infinite ratio, then we just get an error: cannot divide by 0.

Practice exam #7

- Do the row ops to pivot on the 1st column, 1st row, even if this is not the right row or column.

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
-5	0	1	0	-6	0	0	1	15

Practice exam #7

- Do the row ops to pivot on the 1st column, 1st row, even if this is not the right row or column.

x	y	z	a	b	c	d	P	RHS
1	1	1	0	2	0	0	0	11
2	0	1	0	3	1	0	0	12
3	0	1	0	4	0	1	0	13
4	0	1	1	5	0	0	0	14
−5	0	1	0	−6	0	0	1	15

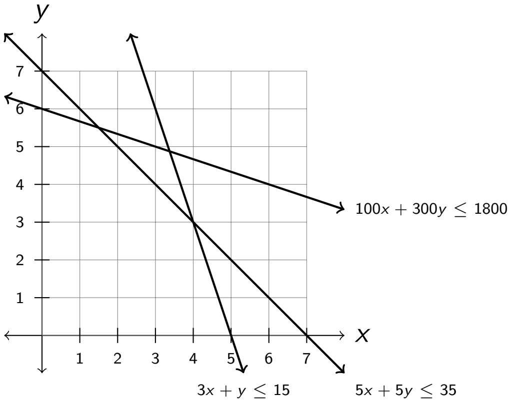
We need to make the matrix be in RREF with the 1st column, 1st row being a ①. This means the 2nd column can no longer have a ①. We need to do one row op per row, but a few of the row ops are easy: $R_1 \to R_1$ is already a 1, we just think of it as ① now.

For the other rows, we want to use R_1 to fix them (make them zero), so that is $R_2 - 2R_1$, $R_3 - 3R_1$, $R_4 - 4R_1$, $R_5 + 5R_1$.

Notice how we chose a bad pivot row and some of the RHS became negative. We'll increase the profit, but break the law.

Practice exam #8

- Maximize $P = 1.10x + 1.20y$ subject to $100x + 300y \leq 1800$, $3x + y \leq 15$, $5x + 5y \leq 35$, $x \geq 0$, $y \geq 0$. Make sure to (1) shade the region, (2) label the corners, (3) label where the maximum occurs and how big it is, and (4) why it must be the maximum.



Practice exam #9

- Vincent is trying to maximize profit using his limited resources. He has four limited resources: 27 tubes of Amarillo paint, 30 tubes of Berry red paint, 12 Canvases, and 30 tubes of Dark blue paint. He only knows how to paint two things: Sunshine and Lollipops, but he can sell as many as he can paint, earning a profit of $10 per Sunshine painting and $12 per Lollipops painting. Each painting requires a few tubes of paint:

	Amarillo	Berry Red	Canvases	Dark Blue	Profit
Sunshine	3	3	1	1	10
Lollipops	1	2	1	3	12
Inventory	27	30	12	30	

Give a recommendation to maximize his profit using only his limited resources:

Practice exam #10

- Vincent has a new painting.

	Amarillo	Berry Red	Canvases	Dark Blue	Profit
Sunshine	3	3	1	1	10
Lollipops	1	2	1	3	12
Rainbows	2	2	1	2	20
Inventory	27	30	12	30	

Hint: When choosing a pivot column, in this case try the book's advice of choosing the most (immediately) profitable column first.

How many of each painting should Vincent produce to maximize his profit using his limited resources?