MA162: Finite mathematics

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Schedule:

- HW 5.3,6.1 are due Fri, November 2nd, 2012
- HW 6.2,6.3 are due Fri, November 9th, 2012
- Exam 3 is Monday, November 12th, 5pm to 7pm in BS107 and BS116

Today we will cover 6.1: Sets

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations

6.1: Life before sets

- We are going to be doing some hard counting problems.
- To make it easier, we need to be able to talk about the things we are counting.
- When we counted money, or acres, or ounces of jamba juice we had variables to denote the number. $x = 5$ acres, or $y = 10$ ounces.
- If you had \$5 in one bank account and \$10 in another, you had \$5+\$10 = \$15 total. The numbers were all that mattered.
- Unfortunately life rarely divides nicely into separate accounts, and numbers cannot describe many of these aspects.

6.1: More than numbers can say

- We are going to be counting more complicated things now.
- If your friend Jimmy says you can borrow their car Monday, Tuesday, and Wednesday, then that is 3 days you've got a car.
- If your friend Timmy says you can borrow their car Tuesday, Thursday, and Friday, then that is 3 days you've got a car.
- How many days total can you borrow a car?

6.1: More than numbers can say

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- If your friend Jimmy says you can borrow their car Monday, Tuesday, and Wednesday, then that is 3 days you've got a car.
- If your friend Timmy says you can borrow their car Tuesday, Thursday, and Friday, then that is 3 days you've got a car.
- How many days total can you borrow a car?
- Well, Monday, Tuesday, Wednesday, Thursday, Friday is five days.
- But $5 \neq 3 + 3$. Numbers are not enough.

6.1: Sets to name the things we are counting

- If we let J be the days Jimmy lets us have the car, then

$$J = \{ \text{ Monday, Tuesday, Wednesday } \}$$

- If we let T be the days Timmy lets us have the car, then

$$T = \{ \text{ Tuesday, Thursday, Friday } \}$$

- The days when at least one of them let us use the car is the **union** of the two sets

$$J \cup T = \{ \text{ Monday, Tuesday, Wednesday, Thursday, Friday } \}$$

- The days when both of them let use the car is the **intersection** of the two sets

$$J \cap T = \{ \text{ Tuesday } \}$$

6.1: More sets

- We can have sets of numbers $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then:
- $A \cup B = \{1, 2, 3, 4, 5\}$
- $A \cap B = \{3\}$
- $A - B = \{1, 2\}$ is the difference, the things in A that are not in B
- We can write down sets in funny ways:
  $A = \{3, 2, 1\} = \{1, 1, 1, 1, 1, 2, 2, 3\}$
- We can describe them in words, "A is the set of positive integers whose square is a one digit number."

6.1: Equality drill

- Two sets are **equal** if they have the same elements.
- $\{1, 2, 3\} \stackrel{?}{=} \{1, 2, 3\}$
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- $\{1, 2, 3\} \stackrel{?}{=} \{1, 2, 2, 3, 3, 3\}$
- $\{1, 2, 3\} \stackrel{?}{=} \{$ positive integers whose square has one digit $\}$
- $\{1, 2, 3\} \stackrel{?}{=} \{$ odd numbers less than 4 $\}$

6.1: Equality drill

- Two sets are **equal** if they have the same elements.
- $\{1,2,3\} = \{1,2,3\}$
  Yes! Exactly the same.
- $\{1,2,3\} \neq \{1,2\}$
  No! Right hand set is missing "3"
- $\{1,2,3\} = \{3,1,2\}$
  Yes! Order does not matter.
- $\{1,2,3\} = \{1,2,2,3,3,3\}$
  Yes! Repeats don't matter.
- $\{1,2,3\} = \{$ positive integers whose square has one digit $\}$
  Yes! Long-winded doesn't matter.
- $\{1,2,3\} \neq \{$ odd numbers less than 4 $\}$
  No! Right hand set is missing "2"

6.1: Union and intersection drill

- $\bigcup$ The **union** includes anything in either, and is big. $\bigcup$
- $\bigcap$ The **intersection** includes only those in both, and is small. $\bigcap$
- $\{1, 2, 3\} \cup \{3, 4, 5\} =$
- $\{1, 2, 3\} \cap \{3, 4, 5\} =$
- $\{1, 2, 3\} \cup \{1\} =$
- $\{1, 2, 3\} \cap \{1\} =$

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- $\{1,2,3\} \cup \{1\} = \{1,2,3\}$
- $\{1,2,3\} \cap \{1\} = \{1\}$

6.1: Difference drill

- $-$ The **difference** keeps the first, but not in the second. $-$
- $\{1, 2, 3\} - \{1\} =$
- $\{1, 2, 3\} - \{2, 3\} =$
- $\{1, 2, 3\} - \{3, 4, 5\} =$
- $\{1, 2, 3\} - \{4, 5, 6\} =$
- $\{1, 2, 3\} - \{1, 2, 3\} =$

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- $\{1,2,3\} - \{2,3\} = \{1\}$
- $\{1,2,3\} - \{3,4,5\} = \{1,2\}$
- $\{1,2,3\} - \{4,5,6\} = \{1,2,3\}$
- $\{1,2,3\} - \{1,2,3\} = \{\}$ The **empty set** containing nothing.

6.1: Laws of sets

- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$, but what about $\{3, 4, 5\} \cup \{1, 2, 3\}$?

6.1: Laws of sets

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- Order of union does not matter

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- $A = \{1,2,3\}$, and $B = \{3,4,5\}$. Compare $A \cap B$ and $A - B$.

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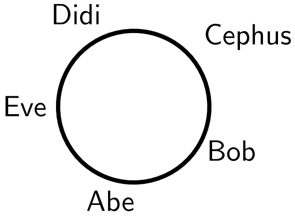
- What about $\{1,2,3\} \cap \{3,4,5\}$ versus $\{3,4,5\} \cap \{1,2,3\}$?
- Both are $\{3\}$.
- $A = \{1,2,3\}$, and $B = \{3,4,5\}$. Compare $A \cap B$ and $A - B$.
- $A \cap B = \{3\}$ and $A - B = \{1,2\}$
- $A = (A \cap B) \cup (A - B)$

- Five friends are playing cards with a standard 52 card deck



- The deck has the following cards:
  A♡ 2♡ 3♡ 4♡ 5♡ 6♡ 7♡ 8♡ 9♡ 10♡ J♡ Q♡ K♡
  A♢ 2♢ 3♢ 4♢ 5♢ 6♢ 7♢ 8♢ 9♢ 10♢ J♢ Q♢ K♢
  A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣
  A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠
- 50 cards have been dealt out, 10 to each person
- Why must one of the suits (♡, ♢, ♣, ♠) be completely dealt out?

6.1: Counting

- Five friends are playing cards with a standard 52 card deck
- 50 cards have been dealt out, 10 each
- Which of the following are true:
- (L) At least two people have at least one clubs ♣
- (R) At least one person has at least two clubs ♣
- (B) Both

6.1: Fancier counting

- Which of the following are true:
- (L) Every player has at least 3 of the same suit
- (R) Some pair of neighbors has at least 6 of the same suit (combined)
- (B) Both
- In “R” it counts if one player has 6 of the same suit by themselves

6.1: Summary

- Today we learned about **sets**, **union**, **intersection**, and **difference**.
- You are now ready to complete 6.1. Better try 6.2 now.
- Make sure to take advantage of office hours