DEPARTMENT OF MATHEMATICS

Ma 162 Third Exam November 12, 2012 (practice)

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	24	
2	18	
3	26	
4	16	
5	16	
Total	100	

NAME: JACK **Section:** 999

Last four digits of Student ID: 9999

(1) Simple Interest: $I = Prt$. Accumulation: $A = P(1 + rt)$.
(2) Compound Interest Accumulation: $A = P(1 + i)^n$. Present value: $P = A(1 + i)^{(-n)}$.
(3) Effective rate: $r_{eff} = (1 + \frac{r}{m})^m - 1$.
(4) Annuity: Sum: $S = R\frac{((1+i)^n-1)}{i}$. Present value: $P = R\frac{(1-(1+i)^{-n})}{i}$.
(5) Set counting: Two sets: $n(A \bigcup B) = n(A) + n(B) - n(A \bigcap B)$
$n(A \bigcup B \bigcup C) = n(A) + n(B) + n(C) - n(A \bigcap B) - n(B \bigcap C) - n(C \bigcap A) + n(A \bigcap B \bigcap C)$.

For each part, you receive points for (1) showing the formula, (2) indicating which numbers go into the formula, (3) getting a number close to the right number, and (4) getting the right number.

1. (a) How much is \$1000 worth after 2 years of 18% APR (1.5% per month) interest compounded monthly?

Compound Interest

$F = \text{Future Value} = P(1+i)^n = \$1000(1.015)^{24}$

P = Present = \$1000
n = time = 24 periods
i = interest rate = 0.015 per period

(b) How much do you need to deposit now to have \$1000 after 2 years of 1% APR interest, compounded monthly?

Ditto

$P = ?$, $F = \$1000$
$n = 24$ periods
$i = 0.01/12$ per period

$F = P(1+i)^n$
$\$1000 = P(1+0.01/12)^{24}$
$P = \$1000 / (1+0.01/12)^{24}$

(c) How much is five years of \$100 monthly payments worth with 12% APR (1% per month) interest? Sinking Fund

$M = \$100$ per period
$i = 0.01$ per period
$n = 60$ periods
$F = M((1+i)^n - 1)/(i)$

$F = \$100\,(1.01^{60} - 1)/0.01$

(d) How much do you need to deposit each month in order to have \$1000 after 2 years of 1% APR interest, compounded monthly? Sinking Fund

$M = ?$
$i = 0.01/12$ per period
$n = 24$ periods
$F = \$1000$

$F = M((1+i)^n - 1)/(i)$
$1000 = M((1+0.01/12)^{24} - 1)/(0.01/12)$
$M = 1000/(((1+0.01/12)^{24} - 1)/(0.01/12))$

(e) How much can you borrow now if you are willing to pay back \$100 per month at 6% APR (0.5% per month) interest, compounded monthly, for seven years? Amortized Loan

$M = \$100$ per period
$i = 0.005$ per period
$n = 84$ periods
$P = M(1 - (1+i)^{(-n)})/(i)$

$P = 100(1 - 1.005^{(-84)})/0.005$

(f) How much does one need to pay each month in order to pay off a \$10,000 loan over 5 years at 6% APR (0.5% per month) interest, compounded monthly? Amortized Loan

$M = ?$
$i = 0.005$
$n = 60$
$P = 10000$

$10000 = M(1 - 1.005^{(-60)})/0.005$
$M = 10000/((1 - 1.005^{(-60)})/0.005)$

For each part, you receive points for (1) explaining your method, (2) explaining your method well, (3) getting a close answer, (4) getting the right answer.

2. Which is the cheaper way to pay the electric bill? The bill is \$100. One does not have the money now, but one will have the money in a month.
How much does each method cost total at the end of the month?
(a) Pay it late. The late fee is \$5.

\$100 + \$5 = \$105

Just a simple fee. Have to pay both bill and fee.

(b) Put it on the credit card, and pay off that purchase and its interest in a month. 36% APR (3% per month).

Total is $\$100(1.03)^1 = \$103$, the interest is \$3 the first (and hopefully only) month.

(c) Pawn an iPhone for a month for \$100. The interest is 1% per month, but there is a \$3.50 processing fee.

Total is $\$100(1.01) + \$3.50 = \$104.50$

(Interest, Fee)

(d) Get a pay-day loan at 416% APR (8% per week) and hope you have the money early.

Total is $\$100(1.08) = \$108$ after 1 week. If you have to borrow again and again each week, it is

$\$100(1.08)^4 =$

(e) (2 points) Which is the cheapest option? Credit Card with \$3 interest (B)

For each part, you receive points for (1) explaining your method, (2) explaining your method well, (3) getting a close answer, (4) getting the right answer.

3. Describe the financial journey of Bob and Carl:

(a) Bob decides to save \$100 per month at 1% per month interest for five years. How much will he have in the account after five years? Sinking Fund

$F = M((1+i)^n - 1)/(i)$ $M = \$100,\ i = 0.01,\ n = 60$

$F = \$100\,(1.01^{60} - 1)/(0.01) = \$8166.97$

(b) After a year, Bob unfortunately realizes the interest rate was really 1% APR. How much is in the account now? Sinking Fund, only one year passed. $i = 0.01/12$ instead

$F = \$100\,((1+0.01/12)^{12} - 1)/(0.01/12)$

$F = \$1205.52$

(c) If he actually wants the amount in part (a) to be there in time (so four more years), how much does he need to start investing per month now?

(b) can sit and earn interest, $\$1205.52(1+0.01/12)^{48} = \$1254.70$

so need an extra $F = 8166.97 - 1254.70 = \$6912.27$ after four years

$M = \$6912.27/(((1+0.01/12)^{48} - 1)/(0.01/12)) = \$141.20$

(d) So at the end of the five years, he has his money. Now he wants to withdraw the money, \$100 per month. Miraculously the interest rate is still 1% APR. How much money is left after five years of withdrawals?

(a) sits and earns interest, $\$8166.97\,(1+0.01/12)^{60} = \$8585.52$

withdrawals sum to a large debt $F = (-\$100)((1+0.01/12)^{60} - 1)/(0.01/12)$

$F = -\$6149.90$. So $\$8585.52 - \$6149.90 = \$2435.62$ left

(e) Carl deposits \$100 per month for two years at 2% APR, compounded monthly. He then withdraws \$100 per month for two years at 2% APR, compounded monthly. How much money is left in his account?

First like (a) $\$100((1+0.02/12)^{24} - 1)/(0.02/12) = \$2446.57$ after two years.

Then (d) $2446.57(1+0.02/12)^{24} - 100((1+0.02/12)^{24} - 1)/(0.02/12) = \$99.76$

↑ Interest on the deposit ↑ Total Debt from withdrawals

almost an entire payment left

(f) (2 points) In plain English, why does Carl still have money left in his account?

While he deposited and withdrew the same amount, that money was left in the account over time, so the amount leftover was the interest.

For each part, you receive points for (1) explaining your method, (2) explaining your method well, (3) getting a close answer, (4) getting the right answer.

4. (a) 40% of those surveyed identified themselves as fiscally conservative. 35% identified themselves as socially conservative. 35% identified themselves as neither. What percentage of those surveyed would identify themselves as fiscally conservative or socially conservative, but not both?

Inclusion Exclusion

$n(F) + n(S) - n(F \cap S) = n(F \cup S)$

$40\% + 35\% - ? = 100\% - 35\%$

$75\% - ? = 65\%$

So 10% identify as both, 65% identify as either, and

$\boxed{55\%} = 65\% - 10\%$ identify as either one or the other, but not both.

(b) What percentage identified as fiscally conservative but not socially conservative?

Inclusion Exclusion

$n(F - S) = n(F) - n(F \cap S)$

$= 40\% - 10\%$

$= \boxed{30\%}$

(c) How many ways can you roll four standard (6-sided) dice without getting any odd numbers on the top of a die?

One die: $n(\{2,4,6\}) = 3$ ways

four dice: $\underset{\text{1st Die}}{3}\ \underset{\text{2nd Die}}{3}\ \underset{\text{3rd Die}}{3}\ \underset{\text{4th Die}}{3} = (3)(3)(3)(3) = \boxed{81}$ ways

Multiplication principle: each roll is independent

(d) If you flip a coin 5 times, how many possible outcomes are there? (For example, HHTTT is one outcome)

$\underset{\text{H or T}}{2}\ \underset{\text{H or T}}{2}\ \underset{\text{H or T}}{2}\ \underset{\text{"}}{2}\ \underset{\text{"}}{2} = 32$ outcomes

Multiplication principle: each coin is independent

Answer 4 of the 5 parts. For each part, you receive points for (1) explaining your method, (2) explaining your method well, (3) getting a close answer, (4) getting the right answer.

5 (a) How many ways can the letters from ACCOUNTANCY be rearranged?

Combinations / Permutations

1 2 3 4 5 6 7 8 9 10 11

$\frac{11}{2}\ \frac{10}{3}\ \frac{9}{2}\ \frac{8}{1}\ \frac{7}{1}\ \frac{6}{2}\ \frac{5}{1}\ \frac{4}{1}\ \frac{3}{1}\ \frac{2}{1}\ \frac{1}{1} = 1663200$ ways

A C C O U N T A N C Y

(b) How many ways can you arrange four letters from GLACIER? Just like the races

1 2 3 4 5 6 7

$\frac{7}{\text{1st Letter}}\ \frac{6}{\text{2nd}}\ \frac{5}{\text{3rd}}\ \frac{4}{\text{4th}} = 840$ ways

(c) How many ways can you arrange all of the letters of GLACIER if you alternate consonants and vowels?

Consonants: G, L, C, R Must go CVCVCVC

Vowels: A, I, E

$\frac{4}{C}\ \frac{3}{V}\ \frac{3}{C}\ \frac{2}{V}\ \frac{2}{C}\ \frac{1}{V}\ \frac{1}{C} = 72$ ways

(d) If you have 5 red beads, 2 green beads, and 4 purple beads, how many different strings of them could you make?

Perm/Comb again. Mult ways if each red is diff (hats). Divide by ways to rearrange reds

$\frac{11}{5}\ \frac{10}{4}\ \frac{9}{3}\ \frac{8}{2}\ \frac{7}{1}\ \Big|\ \frac{6}{2}\ \frac{5}{1}\ \Big|\ \frac{4}{4}\ \frac{3}{3}\ \frac{2}{2}\ \frac{1}{1} = 6930$ ways

(e) If you were making the string of a beads for a friend who cannot distinguish red from green, how many strings could you make that would look different to them?

$\frac{11}{7}\ \frac{10}{6}\ \frac{9}{5}\ \frac{8}{4}\ \frac{7}{3}\ \frac{6}{2}\ \frac{5}{1}\ \Big|\ \frac{4}{4}\ \frac{3}{3}\ \frac{2}{2}\ \frac{1}{1} = 330$ ways

Divide by 7! ways to rearrange red or green.