MA162: Finite mathematics

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November 14, 2012

SCHEDULE:

- Exam 4 is Thursday, December 13th, 6pm to 8pm in:
  CB110 (Sec 001, 002), CB114 (Sec 003, 004), FB200 (Sec 005, 006)
- HW 7A is due Friday, November 23rd, 2012
- HW 7B is due Friday, November 30th, 2012
- HW 7C is due Friday, December 7th, 2012

Today we will cover 7.1: Sample spaces

Final Exam

- Chapter 7: Probability
  - Counting based probability
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  - Empirical probability
  - Conditional probability
- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP

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- If you flip a coin once, it will be heads or tails, but who knows which?
- If you flip a coin 1000 times, it will be heads between 450 and 550 times (with a 99.9% probability).

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- Some things you observe are unique: a sunset, a cloud
- Some things you observe are quite reproducible: when you flip a coin it lands on heads or tails, and each happens about 50% of the time
- An **experiment** is a planned observation of life whose goal is (usually) to confirm a reproducible result
- For example, we might plan an experiment where we flip 10 coins and count how many heads show up.

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- A **sample space** is a list of all the possible outcomes of an experiment
- If we pull one card from the deck, then our sample space can be the set of all 52 (or 54) cards in the deck.
- If we draw five cards from the deck and don’t care about order, then there are $\frac{52}{5}\frac{51}{4}\frac{50}{3}\frac{49}{2}\frac{48}{1} = 2,598,960$ possible outcomes

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- Mhtt $= \{HHH, HHT, HTH, THH\}$ has four sample points in it

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- Not all events are mutually exclusive.
- For instance the event "get a head on the very first try!" is $\{HHH, HHT, HTH, HTT\}$ and so the intersection with "more heads than tails" is $\{HHH, HHT, HTH\}$

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5. (STA291) After actually running the experiment, decide whether your probability calculation reflects reality
6. (STAxxx) Decide how many times to run the experiment before you can decide whether your probability calculation reflected reality

Summary

- We learned the words **experiment**, **sample space**, **event**, and **mutually exclusive**
- HW 7A is two questions. Easy questions.
- HW 7B and 7C are pretty similar to HW 6ABC
- Monday we will cover 7.2: Probability
- Depending on time we might cover it today

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- The event "rolling saved us money" is all those pairs that total to more than 6.
- There are 21 such pairs, and if all pairs are equally likely (the dice are fair), then that is $\frac{21}{36} = \frac{7}{12} \approx 58\%$

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- 16 ways to win, 32 ways total, so $\frac{16}{32} = \frac{1}{2} = 50\%$ chance
- Explicitly:
  HHHHH, HHHHT, HHHTH, HHHTT, HHTTT, HTHHH, HTTTH, HTTTT, THHHH, THHHT, THTTT, TTHHH, TTTHH, TTTHT, TTTTH, TTTTT

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- Some experimenting reveals that about $1/8$th of the time you get 3 heads, $3/8$th of the time you get 2 heads, $3/8$th of the time you get 1 heads, and $1/8$th of the time you get 3 tails.

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- Hence it should be about $\frac{1}{8} + \frac{3}{8} = 50\%$ of the time to get either 1 or 3 heads
- It should be the same for getting an odd number of tails, right? Tails, heads, what is the difference?
- But you either get an odd number of heads, or an odd number of tails, and not both, so each should be about equally likely: 50%

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- However, that's not very likely to happen and quite expensive to plan for.
- If each bulb is independent, that is $(0.1\%)^{700} \approx 0\%$ chance of this happening

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- The odds of none going out is $(99.9\%)^{700} \approx 50\%$, the odds of one are $700 \cdot (0.1\%)(99.9\%)^{699} \approx 35\%$

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- The odds of none going out is $(99.9\%)^{700} \approx 50\%$, the odds of one are $700 \cdot (0.1\%)(99.9\%)^{699} \approx 35\%$
- Total is: $0.844 = 84.4\%$ chance that at most one breaks, so not too bad. Every 6 weeks you'll have a light out and no replacement, but not too bad.

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- What are the odds that 10 is enough?
- The odds of none going out is $(99.9\%)^{7000} \approx 0.1\%$,
  exactly one are $7000 \cdot (0.1\%)(99.9\%)^{6999} \approx 0.6\%$,
  exactly two are $\frac{7000 \cdot 6999}{2} \cdot (0.1\%)^2 (99.9\%)^{6998} \approx 2.2\%$,
  ...

0	1	2	3	4	5	6	7	8	9	10
0.1	0.6	2.2	5.2	9.1	12.7	14.9	14.9	13.0	10.1	7.0

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- Total is: $0.902 = 90.2\%$ chance that at most ten break, so really we're even more certain to be ok now; every 10 weeks we'll be short a bulb.

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- The larger the population, the less extreme the whims of fortune
- This is why insurance is important; the risk to one person is great
- The risk to 10,000 people is quite small, much less than 10,000 times the risk of one

Round table

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- 12 bad out of 30 total is 40% chance for showers (of fists)