MA162: Finite mathematics

Jack Schmidt

University of Kentucky

November 26, 2012

SCHEDULE:

- Exam 4 is Thursday, December 13th, 6pm to 8pm in:
  CB110 (Sec 001, 002), CB114 (Sec 003, 004), FB200 (Sec 005, 006)
- HW 7B is due Friday, November 30th, 2012
- HW 7C is due Friday, December 7th, 2012

Today we will cover 7.5: Conditional probability

Final Exam Breakdown

- Chapter 7: Probability
  - Counting based probability
  - Counting based probability
  - Empirical probability
  - Conditional probability
- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP

7.5: The Punnet square of probability

- Suppose we have the following table of young men and women with and without driver's licenses:

	Yes	No	Total
M	491	9	500
F	486	14	500
T	977	23	1000

- What are the odds a randomly selected person has a driver's license?

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- What are the odds a randomly selected person is female?

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- What are the odds that a randomly selected non-driver is female?

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- Are females less likely to be drivers?

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- Are females less likely to be drivers?
- Probability a female is a driver: $\frac{486}{500} = 97\%$ nearly the same

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- Let's redo this using the language of events:
  - M is the event the chosen person is male
  - F is the event the chosen person is female
  - Y is the event the chosen person has a driver's license
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- We calculated it as $Pr(N \cap F)/Pr(N)$
- We need a name for this calculation, **conditional probability** $Pr(F|N) = Pr(N \cap F)/Pr(N)$ is the probability of F **given** N

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- We want to compare the probabilities of $Pr(A)$ versus $Pr(A|B)$ if they are equal then the events are **independent**

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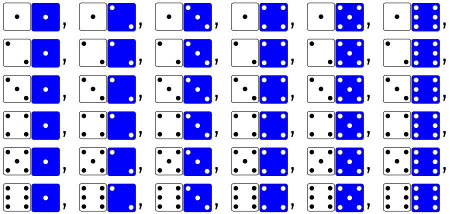
- Just count!



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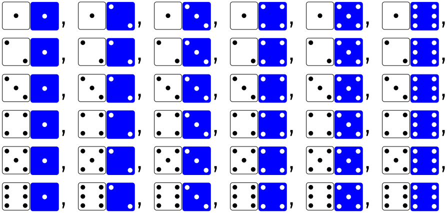

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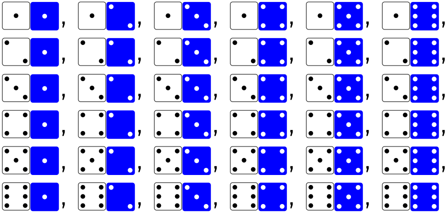

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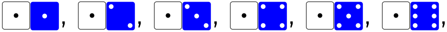

$4/6 \approx 67\%$

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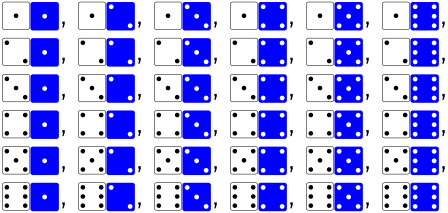

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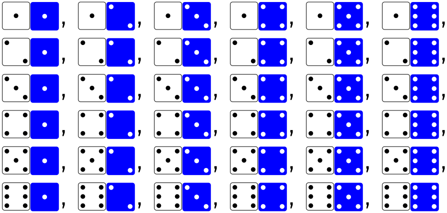

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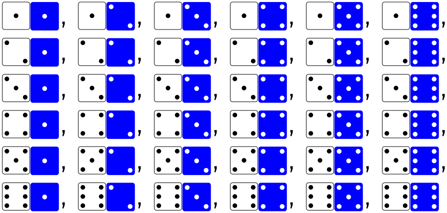

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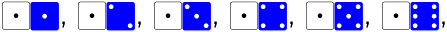

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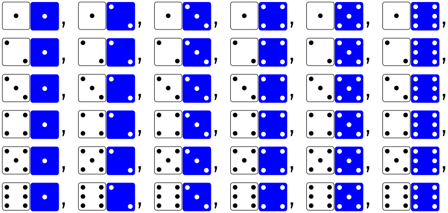

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- Just count!

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- The first die had no effect on the outcome! The two events are said to be **independent**.

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  $85/340 \approx 25\%$
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  "Mostly". The probabilities are not equal, but they are close.

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- Suppose 60% of the time the chip machine gives you your chips, 30% of the time it moves chips around and eats your money, and 10% of the time it gives you double chips,
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- Weighted averages

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- 45%, right?

Reasoning backwards

- Shifty Teddy is spending some time on the gameshow "Who's Gow?" and so you have to use his pal, Shifty Eddy, to run cokes for you. You end up with a coke 30% of the time. How often does he take the money and run?

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- $Pr(F) = 30\%$, and we want to find $Pr(E)$ which we calculated to be 60%, but where do we use the 50%?

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- The coke machine is 50% likely to give you a coke **IF** Eddy gives it the money, so we say $Pr(F|E) = 50\%$, the probability of F **given** E is 50%
- **Bayes's Law:** $Pr(E \cap F) = Pr(F|E) \cdot Pr(E)$ – a weighted average!