MA162: Finite mathematics

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Schedule:

- Exam 4 is Thursday, December 13th, 6pm to 8pm in:
  CB110 (Sec 001, 002), CB114 (Sec 003, 004), FB200 (Sec 005, 006)
- HW 7B is due Friday, November 30th, 2012
- HW 7C is due Friday, December 7th, 2012

Today we will cover harder probability problems, homework-esque and useful

Final Exam Breakdown

- Chapter 7: Probability
  - Counting based probability
  - Counting based probability
  - Empirical probability
  - Conditional probability
- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP

7B #5: Locker search

- For extra security, a high school searches 5 random lockers each school day. If there are 780 students with lockers, what is the probability that a particular student's locker gets searched at least once in a 40 day period?
- What are the odds you get searched on any particular day?

  $\frac{C(1,1)\times C(779,4)}{C(780,5)} = \frac{5}{780}$
- What are the odds you don't get searched that day?

  $1 - \frac{5}{780}$
- What are the odds of that happening 40 times in a row?

  $(1 - \frac{5}{780})(1 - \frac{5}{780}) \cdots (1 - \frac{5}{780}) = (1 - \frac{5}{780})^{40}$
- What are the odds of it failing at least once?

  $1 - (1 - 5/780)^{40}$

7B #6: Two cards

- Standard 52 card deck; two cards drawn without replacement
- First card is A♡, what is the probability the second is A♣?

  $\frac{C(1,1)}{C(51,1)} = \frac{1}{51}$
- More complicated:

  Pr(2nd is A♣| 1st is A♡) = Pr(A♡A♣) / Pr(1st is A♡)

  $\frac{C(2,2)}{C(52,2)} / \frac{C(1,1)C(51,1)}{C(52,2)} = \frac{1}{1326} / \frac{1 \times 51}{1326} = 1/51$
- First card is A♡, what is the probability the second is A♡?

  Silly! Cannot happen. 0% chance. 0 ways out of 51.

7B #10: Deck of cards

- 24 card deck looks like:

  A♡ 2♡ 3♡ 4♡ 5♡ 6♡

  A♢ 2♢ 3♢ 4♢ 5♢ 6♢

  A♣ 2♣ 3♣ 4♣ 5♣ 6♣

  A♠ 2♠ 3♠ 4♠ 5♠ 6♠

- How many possible 3 card hands?

  If we disregard order, it is $C(24, 3) = \frac{(24)(23)(22)}{(3)(2)(1)} = 2024$

- How many have exactly 2 aces?

  We need to choose two aces and another card.

  $C(4, 2) = 6$ ways to choose the aces: ♡♢; ♡♣; ♡♠; ♢♣; ♢♠; ♣♠

  and $C(20, 1) = 20$ ways to choose the other

7B #10: Deck of cards

- $C(24, 3) = 2024$ possible hands
- $C(4, 2) \times C(20, 1) = 6 \times 20 = 120$ hands with exactly two aces
- $C(4, 3) = \frac{(4)(3)(2)}{(3)(2)(1)} = 4$ hands with exactly three aces
- Probablity of getting at least two aces is

$$\frac{120 + 4}{2024} = 6.12\%$$

7B #11: Two hours on a clock

- How many ways to choose two hours on a clock; order doesn't matter; repeats allowed?

  Well $C(12, 2) = 66$ ways to choose two different hours; order doesnt matter

  And $C(12, 1) = 12$ ways to choose one hour twice

  So 78 total ways to choose the hours

- How many ways have the two times close to each other? Maybe within 90 minutes.

  Well the 12 repeats are all 0 minutes apart.

  Each hour also has the "next" hour that works, another 12

- Probability is

$$\frac{12 + 12}{66 + 12} \approx 31\%$$

7C #1: Raffle

- 30 TV tickets, 50 stereo tickets, 520 worthless tickets
- Probability of exactly 1 TV and exactly 1 stereo in 10 tickets?

$$\frac{C(30,1)\times C(50,1)\times C(520,10-1-1)}{C(30+50+520,10)} = \frac{(30)(50)(1256052799067373 45)}{154526905062166886 9640} \approx 12\%$$

- He got exactly 1 TV; probability of no stereo in the other 9 tickets?

$$\frac{C(520,9)}{C(570,9)} \approx 43.5\%$$

- Probability of at least one stereo?

$$1 - \frac{C(520,9)}{C(570,9)} \approx 56.5\%$$

7C #2 and #6: Homework

- 20 things chosen with replacement 3 at a time
- What is probability all 3 are different?

  $\frac{(20)(19)(18)}{(20)(20)(20)} \approx 85.5\%$
- What is probablity at least 2 are same?

  $1 - \frac{(20)(19)(18)}{(20)(20)(20)} \approx 14.5\%$
- What is probability that 3 people born on different days of week?

  $\frac{(7)(6)(5)}{(7)(7)(7)} \approx 61.2\%$
- 4 people?

  $\frac{(7)(6)(5)(4)}{(7)(7)(7)(7)} \approx 35.0\%$
- 8 people?

7C #3 and #7: Quality Assurance

- 60 items, test random half
- Probability that all tested are good, given there is exactly one bad?

  59 good, 1 bad, choose 30

  $\frac{C(59,30)}{C(60,30)} = \frac{1}{2}$ (the bad one is one of the halves!)
- Probability that all tested are good, given there are exactly two bad?

  58 good, 2 bad, choose 30

  $\frac{C(58,30)}{C(60,30)} \approx 24.6\%$ (1/2 chance of first bad being in other half, not quite half chance for the next one)
- Probability that all tested are good, given at least one bad?

  No answer! Need to know more information.

7C #5: study tree

- Homework means 85% to get good grade
- No homework means 14% to get good grade
- If 61% chance of doing homework
- Probability of good grade?

$$(0.61)(0.85) + (1 - 0.61)(0.14) = 57.31\%$$

7C #11: table

- 12 people randomly seated at a table; 2 specific people
- Probability they sit together?

  First guy sits *somewhere*

  Only 2 good chairs left out of 11 chairs possible

  $\frac{2}{11} \approx 18\%$
- What if only 9 chairs, still 12 people?

  Only $\frac{C(2,2) \times C(10,7)}{C(12,9)} = 54.5\%$ chance of them both being seated

  $\frac{2}{8} = 25\%$ chance if they are both seated

  $\frac{C(2,2) \times C(10,7)}{C(12,9)} \times \frac{2}{8} \approx 13.6\%$

Drug test

- A drug test is 98% accurate: out of 100 drug users, 98 will get a positive result, and 2 a negative; out of 100 non-users 98 will get a negative result, and 2 a positive. A company (somehow) knows that exactly 1 of its 100 employees is a drug user, but (somehow) does not know which one.

- An employee is picked at random to be tested, and tests positive. What is the probability that they are the drug user, given that they tested positive? Hint: It is NOT 98%.

- The company wants to be sure, and so tested the employee again. Positive. again. What is the probability that an employee is the drug user, given that they tested positive twice?

Practice exam

- A drug test is 98% accurate: out of 100 drug users, 98 will get a positive result, and 2 a negative; out of 100 non-users 98 will get a negative result, and 2 a positive. A company (somehow) knows that exactly 1 of its 100 employees is a drug user, but (somehow) does not know which one.

- What is the probability that the drug test would correctly report on all 100 employees?

- An employee is picked at random to be tested twice, and tests positive once and negative once. What is the probability an employee is the drug user, given that they tested positive once and negative once?