

MA162: Finite mathematics

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SCHEDULE:

- Exam 4 is Thursday, December 13th, 6pm to 8pm in:
CB110 (Sec 001, 002), CB114 (Sec 003, 004), FB200 (Sec 005, 006)
- HW 7C is due Friday, December 7th, 2012

Today we will cover the harder practice exam problem
(and the practice exam itself should be available on Tuesday)

Final Exam Breakdown

- Chapter 7: Probability
 - Counting based probability
 - Counting based probability
 - Empirical probability
 - Conditional probability
- Cumulative
 - Ch 2: Setting up and reading the answer from a linear system
 - Ch 3: Graphically solving a 2 variable LPP
 - Ch 4: Setting up a multi-var LPP
 - Ch 4: Reading and interpreting answer form a multi-var LPP

7.5: Drug test

- A drug test is 98% accurate:
 - out of 100 drug users, 98 will get a positive result, and 2 a negative;
 - out of 100 non-users, 98 will get a negative result, and 2 a positive.
- A company (somehow) knows that exactly 1 of its 100 employees is a drug user, but (somehow) does not know which one.
- An employee is picked at random to be tested, and tests positive.
- What is the probability that they are the drug user, given that they tested positive?

Hint: It is NOT 98%.

7.5: Drug test

- $P(\text{DU} \mid \text{POS}) = P(\text{DU and POS}) / P(\text{POS})$
- There are two ways to test positive: true positive and false positive

$$\text{True positive: } \frac{1 \text{ user}}{100 \text{ employees}} \times \frac{98 \text{ correct}}{100 \text{ tests}} = 0.0098$$

$$\text{False positive: } \frac{99 \text{ non-users}}{100 \text{ employees}} \times \frac{2 \text{ incorrect}}{100 \text{ tests}} = 0.0196$$

- We want to know how many of those positives are true:

$$\text{True}/(\text{True or False}): \frac{0.0098}{0.0098+0.0196} = \frac{1}{3} = 0.33 = 33\%$$

- So in this company, a 98% accurate test only has a 33% chance of being right when it says “positive”

It has a 99.98% chance of being right when it says “negative”

7.5: Can we be more certain?

- The company wants to be sure, and so tested the employee again.
- Positive, again.
- What is the probability that an employee is the drug user, given that they tested positive twice?

$$\text{True positive: } \frac{1 \text{ user}}{100 \text{ employees}} \times \frac{98 \text{ correct}}{100 \text{ tests}} \times \frac{98 \text{ correct}}{100 \text{ tests}} = 0.009604$$

$$\text{False positive: } \frac{99 \text{ non-users}}{100 \text{ employees}} \times \frac{2 \text{ incorrect}}{100 \text{ tests}} \times \frac{2 \text{ incorrect}}{100 \text{ tests}} = 0.000396$$

$$\text{True}/(\text{True or False}): \frac{0.009604}{0.009604+0.000396} = 0.9604 = 96\%$$

7.5: Can we be more certain?

- The company wants to be sure, and so tested the employee again.
- Positive, again.
- What is the probability that an employee is the drug user, given that they tested positive twice?
- Same idea:

$$\text{True positive: } \frac{1 \text{ user}}{100 \text{ employees}} \times \frac{98 \text{ correct}}{100 \text{ tests}} \times \frac{98 \text{ correct}}{100 \text{ tests}} = 0.009604$$

$$\text{False positive: } \frac{99 \text{ non-users}}{100 \text{ employees}} \times \frac{2 \text{ incorrect}}{100 \text{ tests}} \times \frac{2 \text{ incorrect}}{100 \text{ tests}} = 0.000396$$

$$\text{True}/(\text{True or False}): \frac{0.009604}{0.009604+0.000396} = 0.9604 = 96\%$$

7.5: Another company

- Another company with 43 employees used the test on all of them
- One of them tested positive
- Which is more likely:
 - No employees are drug users
 - One employee is a drug user

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- Another company with 43 employees used the test on all of them
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- Which is more likely:

- No employees are drug users

$$Pr(\text{One pos}|\text{No Users}) = C(43, 1) \times (0.98)^{42} \times 0.02 = 35.96\%$$

- One employee is a drug user

$$Pr(\text{One pos}|\text{One Users}) = C(42, 1) \times (0.98)^{41} \times 0.02^2 + 0.98^{43} = 42.67\%$$

- In statistics class, you learn to find the “most likely” number of users

Practice exam

- A drug test is 98% accurate: out of 100 drug users, 98 will get a positive result, and 2 a negative; out of 100 non-users 98 will get a negative result, and 2 a positive. A company (somehow) knows that exactly 1 of its 100 employees is a drug user, but (somehow) does not know which one.
- What is the probability that the drug test would correctly report on all 100 employees?
- An employee is picked at random to be tested twice, and tests positive once and negative once. What is the probability an employee is the drug user, given that they tested positive once and negative once?