DEPARTMENT OF MATHEMATICS

Ma 162 Final Exam December 13, 2012 (practice)

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If you have no work to support your answer, you will receive no credit. You are graded not on what you know, but on what you write on this exam. Be sure to communicate your understanding, not just write down the final answer.

Problem	Maximum Score	Actual Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

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Last four digits of Student ID: 9999

Dice have six sides: ⚀, ⚁, ⚂, ⚃, ⚄, ⚅. Coins have two sides: H or T.
Make sure to show your work.

1. (a) What is the probability of rolling at least two ⚅ on four dice?

Exactly Two : $C(4,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{(4)(3)}{(2)(1)}\frac{1}{6}\frac{1}{6}\frac{5}{6}\frac{5}{6} = \frac{150}{64}$

Exactly Three : $C(4,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 = \frac{(4)(3)(2)}{(3)(2)(1)}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{5}{6} = \frac{20}{64}$

+ Exactly Four : $C(4,4)\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{(4)(3)(2)(1)}{(4)(3)(2)(1)}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6} = \frac{1}{64}$

$\boxed{\frac{171}{1296}}$

(b) What is the probability of rolling a "double" on three six-sided dice? *(That is, exactly two of the dice are the same; no "triples" allowed. So ⚀⚁⚀ and ⚂⚂⚄ are okay, but ⚀⚀⚀ and ⚁⚂⚄ are not.)*

anything, Same as 1st, anything else

$C(3,2)\,\frac{6}{6}\,\frac{1}{6}\,\frac{5}{6}$

which two are the same

$\boxed{\frac{15}{36}}$

(c) What is the probability of getting "at least four in a row" if you flip a coin seven times? *(That is, four heads with no tails in between, or four tails with no heads in between. So HHHHTTT and THHHHTT and HHHHHHH is okay, but HTHHHTH is not okay.)*

Mutually Exclusive

HHHH*** $\frac{8}{128}$

THHHH** $\frac{4}{128}$

THHHH $\frac{4}{128}$

+ **THHHH $\frac{4}{128}$

4 Heads in a row $\frac{20}{128}$

mut. excl.

4 Heads in a row $20/128$

+ 4 Tails in a row $20/128$

4 in a row $\boxed{40/128}$

(d) What is the probability of getting more than twice as many heads as tails if you flip a coin ten times?

# of Hs	# of Ts	Prob
~~6~~	~~4~~	
7	3	$C(10,3)\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3$
8	2	$C(10,2)\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2$
9	1	$C(10,1)\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1$
+ 10	0	$C(10,0)\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$

$\frac{\frac{(10)(9)(8)}{(3)(2)(1)} + \frac{(10)(9)}{(2)(1)} + \frac{10}{1} + 1}{2^{10}} = \frac{120+45+10+1}{1024} = \boxed{\frac{176}{1024}}$

2. A company wants to determine when to replace its machine belts. It would prefer to replace them before they fail, but would also prefer not to waste them. There is a data-sheet from the belt manufacturer with some failure probabilities recorded.

100% of belts last 30 days or more.
80% of belts last 60 days or more.
50% of belts last 90 days or more.
20% of belts last 120 days or more.

Notice each event contained in previous event

(a) What percentage of belts last between 60 and 90 days?

$$80\% - 50\% = \boxed{30\%}$$

more than 60, but not more than 90

(b) What percentage of belts that have already lasted 60 days will last at least another 30 days?

$$\frac{50\% \ (\geq 90)}{80\% \ (\geq 60)} = \boxed{62.5\%}$$

(c) What is the probability that a belt that has lasted 90 days will fail within the next 30 days?

$$1 - \frac{20\% \ (\geq 120)}{50\% \ (\geq 90)} = \frac{3}{5} = \boxed{60\%}$$

Don't want it to last!

(d) What is the probability that a belt that lasted 60 days will last 120 days or more?

$$\frac{20\% \ (\geq 120)}{80\% \ (\geq 60)} = \boxed{25\%}$$

3. A store is trying out an ad campaign to increase visits. People who have not recently seen the new advertisement have a 2% chance of stopping by the store, but people who have recently seen it have a 30% chance of stopping by.
(a) The first month, only 10% of people have recently seen the ad. What is the probability that someone who stops by the store has seen the ad?

People Who Stop By = PWSB and have seen ad + PWSB and have not

$$= (30\%) \times (10\%) + (2\%) \times (90\%)$$

$$\text{Prob} = \frac{30\% \times 10\%}{30\% \times 10\% + 2\% \times 90\%} = \frac{30}{48} = \boxed{62.5\%}$$

(b) The company decides to spend less money on advertisements, since (in their own words) "so many people have already seen it." The second month, only 1% of people have recently seen the ad. What is the probability that someone who stops by the store has seen the ad?

Same idea

$$\text{Prob} = \frac{30\% \times 1\%}{30\% \times 1\% + 2\% \times 99\%} = \frac{30}{228} = \frac{5}{38} = \boxed{13.2\%}$$

(while closer to 10%, this is probably not good)

(c) What percentage of people ended up stopping by the store during the 1% campaign of part (b)?

Just the denominator:

$$(30\%)(1\%) + (2\%)(99\%) = \boxed{2.28\%}$$

barely more than the original 2%

(d) How does that percentage in part (c) compare to the percentage of people who ended up stopping by the store during the 10% campaign of part (a)?

$$(30\%)(10\%) + (2\%)(90\%) = \boxed{4.8\%}$$

more than double!

4. A company employs people from 3 demographic groups, and after its restructure will either promote, lay off, or retain (at their current position) each employee. The company is concerned that its decisions are independent of the demographic group, and so made the following table:

	Demo1	Demo2	Demo3
Promote	11	7	5
Lay off	21	10	7
Retain	303	163	138
total	335	180	150

	D1	D2	D3
% Prom	3.3	3.9	3.3
% LO	6.3	5.6	4.7
% Ret	90.4	90.5	92
% Tot	100	100	100

(a) What is the probability that an employee will be promoted?

$$\frac{11+7+5}{335+180+150} = \frac{23}{665} = \boxed{3.5\%}$$

(b) What is the probabily that a member of Demo3 will be promoted?

$$\frac{5}{150} = \boxed{3.3\%}$$

(c) What is the probability that an employee will be laid off?

$$\frac{21+10+7}{335+180+150} = \frac{38}{665} = \boxed{5.7\%}$$

(d) What is the probability that an employee that is laid off is in Demo1?

$$\frac{21}{21+10+7} = \frac{21}{38} = \boxed{55.3\%}$$

(e) Which demographic group was treated unfairly and why?

If the were not treated equally, one might say all were treated unfairly.
"independently"

However Demo1 is most likely to complain (least promotions per capita (3.28%) most lay offs per capita (6.3%).

While the bias does not strike me as extreme, it might be worth running it by the Legal department.

5. The Gothic Glee Club is trying to maximize its fund-raiser's profit. They are selling two hand-made candy sculptures: the Santapede and Mrs. Claws. The sculptures require candy canes, marshmallow Santas, and black (like their hearts) licorice. The ingredients and expected profits are given in the following table:

	Candy Cane	Marsh mallow Santas	Black Licorice	Profit
Santapede	8 per	2 per	2 per	\$10 per
Mrs. Claws	4 per	2 per	4 per	\$12 per
Inventory	96	30	52	

In plain English, give a recommendation to the GGC to maximize their profit:

Make 4 Santapedes and 11 Mrs. Claws. This gives \$172 profit, uses all marshmallows and licorice, but leaves 20 candy canes as door prizes.

Now justify it with mathematics:

GGC wants to maximize $P = 10x + 12y$

subject to:

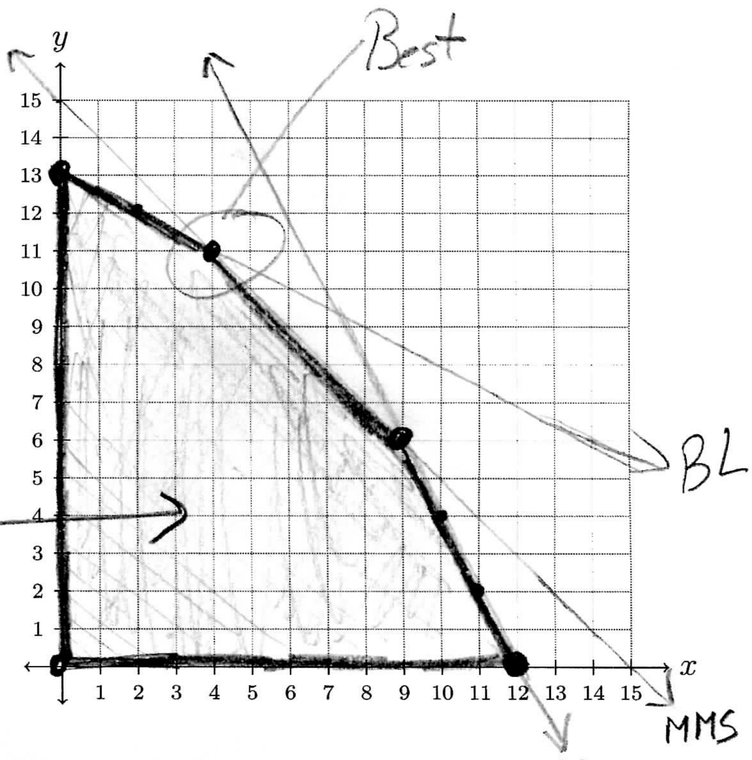
$8x + 4y \le 96$ (don't run out of CC)

$2x + 2y \le 30$ (" " " MMS)

$2x + 4y \le 52$ (" " " BL)

$x \ge 0,\ y \ge 0$ (we sell, not buy)

We graph all possible ("feasible") business decisions



The best (and worst) decisions are at the corners:

x	y	P
0	0	0
0	13	156
4	11	172
9	6	162
12	0	120

— Best

This will use all BL and MMS, but leave some CC leftover

$(4)(8) + (11)(4) = 76$, so 20 CC leftover.