DEPARTMENT OF MATHEMATICS

Ma 162 Final Exam December 13, 2012 (practice)

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If you have no work to support your answer, you will receive no credit. You are graded not on what you know, but on what you write on this exam. Be sure to communicate your understanding, not just write down the final answer.

Problem	**Maximum Score**	**Actual Score**
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

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Last four digits of Student ID: 9999

Dice have six sides: ⚀, ⚁, ⚂, ⚃, ⚄, ⚅. Coins have two sides: H or T.
Make sure to show your work.

1. (a) What is the probability of rolling at least two ⚅ on four dice?

Possible rolls: $6 \cdot 6 \cdot 6 \cdot 6 = 1296$

Ways to get exactly zero: $5 \cdot 5 \cdot 5 \cdot 5 = 625$

Ways to get exactly one: $1 \cdot 5 \cdot 5 \cdot 5 \times 4 = 500$ (6 could be on any of them)

At least two = $1296 - 625 - 500 = 171$

Probability: $171/1296 \approx 13.2\%$

(b) What is the probability of rolling a "double" on three six-sided dice? *(That is, exactly two of the dice are the same; no "triples" allowed. So ⚀⚁⚀ and ⚂⚂⚄ are okay, but ⚀⚀⚀ and ⚁⚂⚃ are not.)*

Ways to get a double: $6 \cdot 1 \cdot 5 \cdot 3 = 90$

- 6: First die
- 1: Second is same
- 5: other value (to avoid triple)
- 3: other value could be in one of three places

Possible rolls: $6 \cdot 6 \cdot 6 = 216$

Probability: $90/216 = 15/36 \approx 41.6\%$

(c) What is the probability of getting "at least four in a row" if you flip a coin seven times? *(That is, four heads with no tails in between, or four tails with no heads in between. So HHHHTTT and THHHHTT and HHHHHHH is okay, but HTHHHTH is not okay.)*

Possible flips $= 2^7 = 128$

Assume four heads in a row.

HHHH*** Ways to do last three: $2 \times 2 \times 2 = 8$

*HHHH** Again 8 ways, but we have already counted ones where flip 1 = H, so THHHH** has $2 \times 2 = 4$

**HHHH* 8 ways, but already counted ones where flip 2 = H, so *THHHH* has $2 \times 2 = 4$

***HHHH 8 ways, but already counted ones where flip 3 = H, so **THHHH has $2 \times 2 = 4$.

$8 + 4 + 4 + 4 = 20$. Double this to include similar cases for T.

Probability: $40/128 \approx 31\%$

(d) What is the probability of getting more than twice as many heads as tails if you flip a coin ten times?

Possible coin flips: $2^{10} = 1024$

10 heads, 0 tails: 1

9 heads, 1 tail: $C(10,1) = 10$

8 heads, 2 tails: $C(10,2) = 10!/8!2! = \frac{10 \cdot 9}{2} = 45$

7 heads, 3 tails: $C(10,3) = 10!/7!3! = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120$

6 heads, 4 tails – Not more than twice as many!

$120 + 45 + 10 + 1 = 176$

Probability: $\frac{176}{1024} \approx 17.1\%$

2. A company wants to determine when to replace its machine belts. It would prefer to replace them before they fail, but would also prefer not to waste them. There is a data-sheet from the belt manufacturer with some failure probabilities recorded.

100% of belts last 30 days or more.
80% of belts last 60 days or more.
50% of belts last 90 days or more.
20% of belts last 120 days or more.

(a) What percentage of belts last between 60 and 90 days?

$80\% - 50\% = \boxed{30\%}$

(b) What percentage of belts that have already lasted 60 days will last at least another 30 days?

$$\frac{P(\text{lasts 90 days or more})}{P(\text{lasted 60 days})} = \frac{50\%}{80\%} = \boxed{62.5\%}$$

(c) What is the probability that a belt that has lasted 90 days will fail within the next 30 days?

$$\text{Probability that it WON'T fail} = \frac{P(\text{lasts 120 days})}{P(\text{lasted 90 days})} = \frac{20\%}{50\%} = \frac{2}{5} = 40\%$$

$$\text{Probability that it WILL fail} = 1 - 40\% = \boxed{60\%}$$

(d) What is the probability that a belt that lasted 60 days will last 120 days or more?

$$\frac{P(\text{lasts 120 days})}{P(\text{lasted 60 days})} = \frac{20\%}{80\%} = \boxed{25\%}$$

3. A store is trying out an ad campaign to increase visits. People who have not recently seen the new advertisement have a 2% chance of stopping by the store, but people who have recently seen it have a 30% chance of stopping by.

(a) The first month, only 10% of people have recently seen the ad. What is the probability that someone who stops by the store has seen the ad?

A = People have seen the ad $P(A) = 10\%$

B = People Stop By

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{10\% \times 30\%}{\underbrace{90\% \times 2\%}_{\text{haven't seen it}} + \underbrace{10\% \times 30\%}_{\text{have seen it}}} = \frac{.03}{.018 + .03} = \boxed{62.5\%}$$

(b) The company decides to spend less money on advertisements, since (in their own words) "so many people have already seen it." The second month, only 1% of people have recently seen the ad. What is the probability that someone who stops by the store has seen the ad?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1\% \times 30\%}{\underbrace{99\% \times 2\%}_{\text{haven't seen it}} + \underbrace{1\% \times 30\%}_{\text{have seen it}}} = \frac{.003}{.0198 + .003} = \frac{.003}{.0228} = \boxed{13.2\%}$$

(c) What percentage of people ended up stopping by the store during the 1% campaign of part (b)?

during 1%

$$P(B) = 99\% \times 2\% + 1\% \times 30\% = .0228 = \boxed{2.28\%}$$

Not much more than if the ad hadn't been run.

(d) How does that percentage in part (c) compare to the percentage of people who ended up stopping by the store during the 10% campaign of part (a)?

during 10%

$$P(B) = 90\% \times 2\% + 10\% + 30\% = .048 = \boxed{4.8\%}$$

This is more than double the amount of when they spent less.

4. A company employs people from 3 demographic groups, and after its restructure will either promote, lay off, or retain (at their current position) each employee. The company is concerned that its decisions are independent of the demographic group, and so made the following table:

	Demo1	Demo2	Demo3
Promote	11	7	5
Lay off	21	10	7
Retain	303	163	138
Total	335	180	150

(a) What is the probability that an employee will be promoted?

$$\frac{\text{Total promoted}}{\text{Employees}} = \frac{11+7+5}{335+180+150} = \frac{23}{665} \approx \boxed{3.46\%}$$

(b) What is the probabily that a member of Demo3 will be promoted?

$$\frac{5}{150} \approx \boxed{3.33\%}$$

(c) What is the probability that an employee will be laid off?

$$\frac{\text{Total laid off}}{\text{Employees}} = \frac{21+10+7}{335+180+150} = \frac{38}{665} \approx \boxed{5.71\%}$$

(d) What is the probability that an employee that is laid off is in Demo1?

$$P(\text{Demo1} \mid \text{laid off}) = \frac{P(\text{laid off and in Demo1})}{P(\text{laid off})} = \frac{21/665}{38/665} = \frac{21}{38} \approx \boxed{55.26\%}$$

(e) Which demographic group was treated unfairly and why?

The answer to (d) makes the layoffs seem unfair to Demo1.
They only represent 50.3% of the employee population and are over-represented in layoffs.
Also, the probability that a promoted employee is in Demo1 is 47.8%, so they are underrepresented in promotions compared to their % in employee population.

5. The Gothic Glee Club is trying to maximize its fund-raiser's profit. They are selling two hand-made candy sculptures: the Santapede and Mrs. Claws. The sculptures require candy canes, marshmallow Santas, and black (like their hearts) licorice. The ingredients and expected profits are given in the following table:

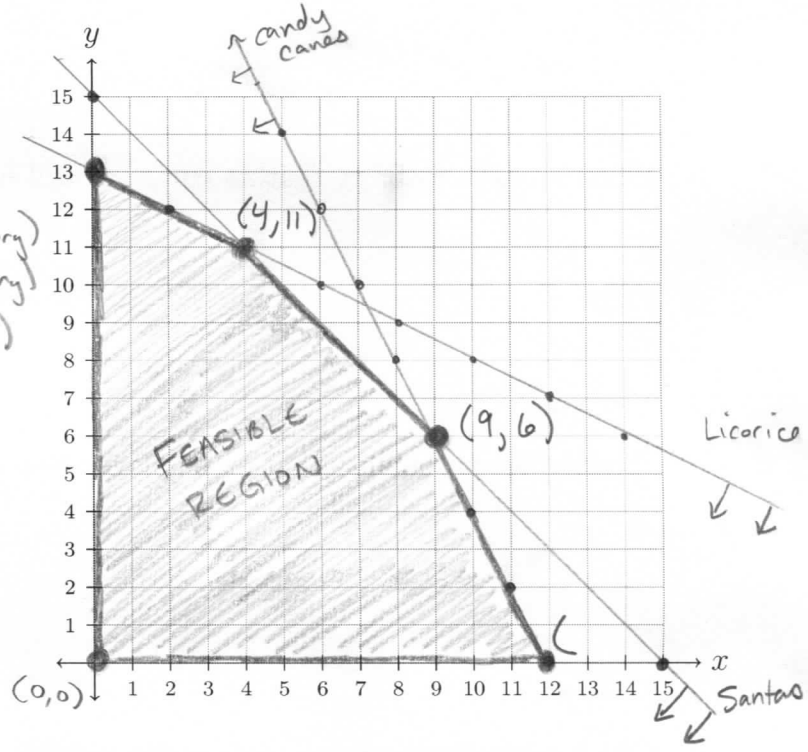
	Candy Cane	Marsh mallow Santas	Black Licorice	Profit
Santapede	8 per	2 per	2 per	\$10 per
Mrs. Claws	4 per	2 per	4 per	\$12 per
Inventory	96	30	52	

In plain English, give a recommendation to the GGC to maximize their profit:
To maximize profit, make 4 Santapedes and 11 Mrs Claus's. This will make \$172 profit. All Santas and licorice will be used up, but there will be 20 candy canes leftover. This will allow for some breakage and eating during construction.

Now justify it with mathematics:

x = # of Santapedes
y = # of Mrs. Claws

maximize $10x + 12y$
s.t. $8x + 4y \le 96$ (candy cane inventory)
$2x + 2y \le 30$ (Santa inventory)
$2x + 4y \le 52$ (licorice inventory)
$x, y \ge 0$ (sanity)



Candy Cane:
$y \le -2x + 24$

Santas:
$y \le -x + 15$

Licorice
$y \le -\frac{1}{2}x + 13$

Check profit at all corners to find maximum:

x	y	P = 10x + 12y	
0	0	0	
4	11	172	← MAX PROFIT
0	13	156	
9	6	162	
12	0	120	

Find corners:

Licorice vs Santa
$(-1)\ 2x + 2y = 30$
$2x + 4y = 52$
$2y = 22$
$y = 11$
$2x + 2(11) = 30$
$x = 4$
(4, 11)

Candy cane vs Santa
$(-1)\ y = -2x + 24$
$y = -x + 15$
$0 = x - 9$
$9 = x$
$y = -9 + 15 = 6$
(9, 6)

Leftovers?
Candy canes
$8(4) + 4(11) = 76$
$96 - 76 = 20$ leftover

Santas
$2(4) + 2(11) = 30$ (all used)

Licorice
$2(4) + 4(11) = 52$ (all used)