MA162: Finite mathematics

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Schedule:

- HW 0A due Friday, Jan 11, 2013 (Late; worth half credit)
- HW 1.1-1.4 due Friday, Jan 18, 2013
- HW 2.1-2.2 due Friday, Jan 25, 2013
- HW 2.3-2.4 due Friday, Feb 01, 2013
- Exam 1, Monday, Feb 04, 2013, from 5pm to 7pm

Today we cover more linear models (1.3-1.4), specifically Cost-Revenue-Profit and Supply-Demand.

Ch 1.3: Example 2: Cost, Revenue, Profit

- You can sell corn at $17 per bushel
- It costs you $4 per bushel to make it
- Before you even make a single bushel of corn, you are $1001 in debt
- How much are you in debt to make 10 bushels?
- How much do you sell those 10 bushels for?
- How does that work out for you?

Ch 1.3: Example 2: Cost, Revenue, Profit

- Well your costs are easy: \$1001 plus \$4 per bushel

$$C(x) = 1001 + 4x$$

- Your revenue is easy: \$17 per bushel

$$R(x) = 17x$$

- So profit is easy, you start \$1001 in the hole, and make \$13 per bushel

$$P(x) = -1001 + 13x$$

Ch 1.3: Example 2: Cost, Revenue, Profit

- At 10 bushels, you've made \$170 but spent \$1041, so you are \$871 in debt
- At 20 bushels, you've made \$340 but spent \$1081, so you are \$741 in debt
- Every additional 10 bushels gets you an additional \$130 closer to breaking even
- \$741/\$130 is about 5.7 so probably need another 57 bushels, let's check:
- At 77 bushels, you've made \$1309 but spent \$1309, so you've just broken even
- At 100 bushels, you've made \$1700 but spent \$1401, so you are \$299 ahead
- $(100 - 77)(13) = (23)(13) = 299$. Not a coincidence.

Ch 1.3: Example 2: Cost, Revenue, Profit

- **Marginal cost** is \$4 per bushel
- **Fixed cost** is \$1001
- **Marginal revenue** is \$17 per bushel
- **Marginal profit** is \$13 per bushel
- **Break-even production** is 77 bushels

Ch 1.3: Did we understand it?

- Fixed and marginal cost (new product)
- 20 cost \$200, 25 cost \$220, how much do 30 cost?

(Left) \$300

(Right) \$240

(Both) \$225

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- Discuss with your neighbors, because you'll explain it to us next

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- 20 cost \$200, 25 cost \$220, how much do 30 cost?

(Left) \$300

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- Discuss with your neighbors, because you'll explain it to us next
- Now explain it to us, especially someone who changed their mind.

Ch 1.3: Did we understand it?

- 20 cost \$200, 25 cost \$220, how much do 30 cost?

(Left) \$300 – This assumes each one costs \$10,
but then 25 should have costed \$250

(Right) \$240 – 5 more costed \$20 more,
so another 5 costs another \$20

(Both) 5 more costs \$5 more? Life isn't that simple

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- So **Marginal cost** is \$20 per 5, or \$4 each

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- So **Marginal cost** is \$20 per 5, or \$4 each
- So **fixed cost** is \$120

Ch 1.3: Do we understand it now?

- 50 cost \$500, 100 cost \$700, how much do 75 cost?

(Left) \$750

(Right) \$900

(Both) \$600

Ch 1.3: Do we understand it now?

- 50 cost \$500, 100 cost \$700, how much do 75 cost?

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(Right) \$900

(Both) \$600

- 50 more cost \$200 more, so 25 more only costs \$100 more
  (Both) \$600

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- 50 cost \$500, 100 cost \$700, how much do 75 cost?

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(Right) \$900

(Both) \$600

- 50 more cost \$200 more, so 25 more only costs \$100 more
  (Both) \$600

- **Marginal cost** is \$4 each

Ch 1.3: Do we understand it now?

- 50 cost \$500, 100 cost \$700, how much do 75 cost?

(Left) \$750

(Right) \$900

(Both) \$600

- 50 more cost \$200 more, so 25 more only costs \$100 more
  (Both) \$600

- **Marginal cost** is \$4 each

- **Fixed cost** is \$300,
  since \$4 each for 50 is only \$200, not \$500

Ch 1.4: Intersecting lines: Examples 2-5

- The break-even point is when the **revenue equals the cost**
- $R(x) = C(x)$
- To solve $17x = 1001 + 4x$, move the xs over to get

$$13x = 1001 \qquad x = 1001/13 = 77$$

- A pessimistic phrasing is when the **profit is zero**
- $P(x) = 0$
- To solve $-1001 + 13x = 0$, move the 1001 over to get

$$13x = 1001 \qquad x = 1001/13 = 77$$

Ch 1.3: Example 3: Supply function

- All else being equal, more people are willing to supply at a higher price
- $x = 40p + 100$ describes the number x of bushels people are willing to supply at a price p in dollars per bushel.

  The 40 has units "bushels per (dollar per bushel)" and the 100 has units "bushels"
- How many bushels would be supplied at \$4 per bushel?
- How many bushels would be supplied at \$5 per bushel?
- How many bushels would be supplied at \$17 per bushel?
- How many extra bushels are supplied for every extra dollar per bushel in price?

Ch 1.3: Example 3: Demand function

- Demand is exactly the same, but is controlled by the buyers.
- The demand is 1170 bushels at \$4 per bushel
- The demand drops to 0 bushels at \$17 per bushel
- In the middle, we assume a "linear demand curve" or model
- How much did the demand drop?
- How much did the price increase?
- How much did demand drop per dollar of price increase?
- What would the demand at \$5 per bushel be?

Ch 1.4: Example 6-7: Market equilibrium

- How much is supplied at \$4 per bushel? How much is demanded? What is the shortfall?
- How about at \$5? What is the shortfall?
- How much does the shortfall decrease per dollar-per-bushel increase in price?
- When does the shortfall drop to 0?

Ch 1.4: Worked out

- At \$4, we calculated supply 260 bushels, demand was 1170 bushels, shortfall is 910
- At \$5, we calculated supply was 300 bushels, demand was 1080, shortfall is 780
- Each dollar the supply increases by 40 and the demand drops by 90, so the shortfall is dropping by $40 + 90 = 130$ bushels
- At \$4 the shortfall is 910, so we need to raise the price by another $910/130 = 7$ dollars per bushel to drop the shortfall to 0
- That is $\$4 + \$7 = \$11$ per bushel at market equilibrium
- Supply is $40(11) + 100 = 540$ and Demand is $1170 - 90(11 - 4) = 1170 - 90(7) = 540$

Ch 1.4: Example 6-7: Market equilibrium

- In a rational, free market, the demand (number of items bought) equals the supply (number of items sold)
- On the exam, a problem like this requires you to:
  - find the supply equation
  - find the demand equation
  - set them equal to each other
  - solve for the **equilibrium quantity**
  - substitute back in for the **equilibrium price** (or vice versa)