MA162: Finite mathematics

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SCHEDULE:

- HW 1.1-1.4 due Friday, Jan 18, 2013 (Late)
- HW 2.1-2.2 due Friday, Jan 25, 2013 (Late)
- HW 2.3-2.4 due Friday, Feb 01, 2013
- Exam 1, Monday, Feb 04, 2013, from 5pm to 7pm
- HW 3.1 due Friday, Feb 08, 2013

Today we cover 2.4: matrix arithmetic.

Production matrix

- Many businesses convert "raw" materials into finished goods

- FroYo-Palooza converts weird chemical mixtures into frozen yogurt
  To make 8 ounces of their standard flavors, they use the following number of ounces of stuff:

	Vanilla	Tart	Mango	Surprise
White stuff	7 oz	6 oz	5 oz	4 oz
Clear stuff	1 oz	1 oz	1 oz	1 oz
Yellow stuff	0 oz	1 oz	0 oz	2 oz
Orange stuff	0 oz	0 oz	2 oz	1 oz

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- What if they switch to making 16 oz FroYos?
  What would the table look like?

Inventory and delivery matrix

- Many businesses resell items kept on shelves at multiple locations
- Wally's World of Weird Socks has 3 locations and 4 types of socks

Inventory	Argyle	Tie-Dye	Fish-net	Toe-socks
Lexington	20	20	5	20
Frankfort	10	20	10	20
Cincinnati	20	20	20	20

Inventory and delivery matrix

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- Wally's World of Weird Socks has 3 locations and 4 types of socks

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Lexington	20	20	5	20
Frankfort	10	20	10	20
Cincinnati	20	20	20	20

- Occasionally people buy socks, so new socks must be delivered

Delivery	Argyle	Tie-Dye	Fish-net	Toe-socks
Lexington	2	2	1	2
Frankfort	1	2	1	2
Cincinnati	2	2	2	2

Inventory and delivery matrix

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- Wally's World of Weird Socks has 3 locations and 4 types of socks

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- Occasionally people buy socks, so new socks must be delivered

Delivery	Argyle	Tie-Dye	Fish-net	Toe-socks
Lexington	2	2	1	2
Frankfort	1	2	1	2
Cincinnati	2	2	2	2

- What would a sales record look like?

Inventory and delivery matrix

- Many businesses resell items kept on shelves at multiple locations
- Wally's World of Weird Socks has 3 locations and 4 types of socks

Inventory	Argyle	Tie-Dye	Fish-net	Toe-socks
Lexington	20	20	5	20
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Cincinnati	20	20	20	20

- Occasionally people buy socks, so new socks must be delivered

Delivery	Argyle	Tie-Dye	Fish-net	Toe-socks
Lexington	2	2	1	2
Frankfort	1	2	1	2
Cincinnati	2	2	2	2

- What would a sales record look like?
- How does one combine the inventory, sales, and delivery tables?

2.4: Matrix arithmetic

- We saved time and worked more efficiently by converting systems of equations to matrices
- We treated each row of a matrix like a single (fancy) number,
- We added rows, subtracted rows, and multiplied rows by numbers
- Now we learn to treat entire matrices as (very fancy) numbers
- Today we will **add**, **subtract**, **multiply by numbers**, and **multiply**
- Next week we will divide; in chapter 3 we will solve real problems

2.4: Matrix size

- A matrix is a rectangular array of numbers, like a table
- A matrix has a **size**: the number of **rows** and **columns**
- A 2×3 matrix has 2 rows, and 3 columns like:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- A 1×4 matrix has 1 row and 4 columns like:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

- A 1×1 matrix has 1 row and 1 column like:

$$\begin{bmatrix} 19 \end{bmatrix}$$

2.4: Matrix equality

- Two matrices are equal if they have the same size, and the same numbers in the same place

- If these two matrices are equal,

$$\begin{bmatrix} 1 & x \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} y & 2 \\ 3 & 4 \end{bmatrix}$$

  then $x = 2$ and $y = 1$

- None of these matrices are equal to each other:

$$\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

2.4: Matrix addition

- We can add matrices if they are the same size by adding entry-wise:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} + \begin{bmatrix} 21 & 22 \\ 23 & 24 \end{bmatrix} = \begin{bmatrix} 11+21 & 12+22 \\ 13+23 & 14+24 \end{bmatrix} = \begin{bmatrix} 32 & 34 \\ 36 & 38 \end{bmatrix}$$

- Big matrices are no harder, just more of the same:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} + \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \begin{bmatrix} 22 & 24 & 26 & 28 \\ 30 & 32 & 34 & 36 \\ 38 & 40 & 42 & 44 \end{bmatrix}$$

- Different shaped matrices are not added together:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} + \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \textbf{nonsense; undefined}$$

2.4: Matrix subtraction

- We can subtract matrices if they are the same size:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} - \begin{bmatrix} 21 & 22 \\ 23 & 24 \end{bmatrix} = \begin{bmatrix} 11-21 & 12-22 \\ 13-23 & 14-24 \end{bmatrix} = \begin{bmatrix} -10 & -10 \\ -10 & -10 \end{bmatrix}$$

- Big matrices are no harder, just more of the same:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} - \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \begin{bmatrix} -20 & -20 & -20 & -20 \\ -20 & -20 & -20 & -20 \\ -20 & -20 & -20 & -20 \end{bmatrix}$$

- Different shaped matrices are not subtracted from one another:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} - \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \textbf{nonsense; undefined}$$

2.4: Scalar multiplication

- We can multiply a matrix by a number (a **scalar**):

$$5 \cdot \begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} = \begin{bmatrix} 5 \cdot 11 & 5 \cdot 12 \\ 5 \cdot 13 & 5 \cdot 14 \end{bmatrix} = \begin{bmatrix} 55 & 60 \\ 65 & 70 \end{bmatrix}$$

- Big matrices are no harder, just more of the same:

$$3 \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 & 12 \\ 15 & 18 & 21 & 24 \\ 27 & 30 & 33 & 36 \end{bmatrix}$$

- There is no restriction on size of the matrix,
  but remember we aren't multiplying two matrices yet:

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = ???$$

Production and orders

- FroYo-Palooza converts weird chemical mixtures into frozen yogurt To make 8 ounces of their standard flavors, they use the following number of ounces of stuff:

	Vanilla	Tart	Mango	Surprise
White stuff	7 oz	6 oz	5 oz	4 oz
Clear stuff	1 oz	1 oz	1 oz	1 oz
Yellow stuff	0 oz	1 oz	0 oz	2 oz
Orange stuff	0 oz	0 oz	2 oz	1 oz

- They have three FroYo machines that fill the following orders:

	Front	Middle	Back
Vanilla	4	6	3
Tart	2	1	1
Mango	2	1	3
Surprise	1	2	4

- How much white stuff does the front machine use?

2.5: Matrix-matrix multiplication

- Matrix-matrix multiplication can be defined several ways
- Only one way is particularly useful to us in this class
- A simple example: We want to write down

$$\begin{aligned} 1x + 2y &= 3 \\ 4x + 5y &= 6 \end{aligned}$$

- Using our multiplication this becomes:

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

- Cleanly separates the variables and the numbers,
  keeps the $+$ and $=$ signs, so lets us be more flexible

2.5: Matrix-matrix multiplication

- To find the **top-left** entry of the product, we multiply the **top** row by the **left** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & ? \\ ? & ? \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} 58 & ? \\ ? & ? \end{bmatrix}$$

2.5: Matrix-matrix multiplication

- To find the **top-right** entry of the product, we multiply the **top** row by the **right** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & 1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12 \\ ? & ? \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & 8 + 20 + 36 \\ ? & ? \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ ? & ? \end{bmatrix}$$

2.5: Matrix-matrix multiplication

- To find the **bottom-left** entry of the product, we multiply the **bottom** row by the **left** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & 1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12 \\ \\ 4 \cdot 7 + 5 \cdot 9 + 6 \cdot 11 & ? \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & 8 + 20 + 36 \\ \\ 28 + 45 + 66 & ? \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ \\ 139 & ? \end{bmatrix}$$

2.5: Matrix-matrix multiplication

- To find the **bottom-right** entry of the product, we multiply the **bottom** row by the **right** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & 1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12 \\ 4 \cdot 7 + 5 \cdot 9 + 6 \cdot 11 & 4 \cdot 8 + 5 \cdot 10 + 6 \cdot 12 \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & 8 + 20 + 36 \\ 28 + 45 + 66 & 32 + 50 + 72 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$