

MA162: Finite mathematics

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SCHEDULE:

- Exam PDFs on mathclass.org; number grades will go up today
- HW 3.1 (Late)
- HW 3.2-3.3 due Friday, Feb 15, 2013
- HW 4.1 due Friday, Feb 22, 2013
- HW 2.5-2.6 due Friday, Mar 01, 2013
- Exam 2, Monday, Mar 04, 2013, from 5pm to 7pm

Today we will cover 3.2: setting up linear programming problems

3.2: Linear programming problems

- An LPP has three parts:
 - The variables (the business decision to be made)
 - The inequalities (the laws, constraints, rules, and regulations)
 - The objective (maximize profit, minimize cost)
- Setting up the problem will be **your job!**
- Reading the answer will be **your job!**
- The middle part is on the exam and you can do it!

3.2: Example 1. Production problem (1/2)

- Ace Cozies is a small company producing two products:
 - Bottle cozies with sippy cup tops
 - Phone cozies with shatter resistant case
- It uses modern micro-manufacturing techniques including its:
 - MakerBot computer aided 3D printer
 - KnitBot-2010 computer controlled knitting machine
 - Assembly crew (people)

3.2: Example 1. Production problem (2/2)

- Each Bottle Cozy realizes the company a profit of \$10
Each Phone Cozy realizes the company a profit of \$12
- Each item requires a certain amount of time (in minutes):

	3D Printer	KnitBot	Crew
Bottle	26	60	20
Phone	62	30	40

- Time is short: Each day the company can only run the 3D printer 5 hours, the KnitBot 4 hours, and the crew 4 hours.
- The union is strong: The total machine time can only be three times as much as the human time
- How can you maximize profit without destroying the machines or ticking off the union?

3.2: Example 1. Setting it up (1/3)

- What do you actually have control over? What are the variables?

3.2: Example 1. Setting it up (1/3)

- What do you actually have control over? What are the variables?
- Maybe you should start by deciding how many bottle cozies and how many phone cozies to make.
- The manager (you) sets the **Production Goals** in order to maximize profit legally
- We use **variables** to describe our decision:
 - X = the number of bottle cozies to make each day
 - Y = the number of phone cozies to make each day

3.2: Example 1. Setting it up (2/3)

- What constraints do we operate under?

$$26X + 62Y \leq 300 \quad (\text{3D printer time})$$

$$60X + 30Y \leq 240 \quad (\text{KnitBot time})$$

$$20X + 40Y \leq 240 \quad (\text{Human time})$$

$$26X - 28Y \leq 0 \quad (\text{Union req.})$$

- Sanity: $X \geq 0$, $Y \geq 0$ (standard inequalities)

- Union requirement:

Machine time is $26X + 60X + 62Y + 30Y = 86X + 92Y$ and

Human time times three is $3(20X + 40Y) = 60X + 120Y$

So requirement is $86X + 92Y \leq 60X + 120Y$, or

$$26X - 28Y \leq 0$$

3.2: Example 1. Setting it up (3/3)

- Ok, no problem. I have the answer. $X = 0$ and $Y = 0$. No rules are broken!
 - We need a **goal**. We need an **objective**:
 - **Maximize** the profit $P = 10X + 12Y$
-
- We can do a lot better than $X = 0$ and $Y = 0$ (with $P = 0$)
 - Even $X = 1$ and $Y = 1$ is better! ($P = 22$ and no rules broken)

3.2: Example 1. Summary

- **Variables:**

X = the number of bottle cozies to make each day

Y = the number of phone cozies to make each day

- **Constraints:**

$$26X + 62Y \leq 300 \quad (\text{3D printer time})$$

$$60X + 30Y \leq 240 \quad (\text{KnitBot time})$$

$$20X + 40Y \leq 240 \quad (\text{Human time})$$

$$26X - 28Y \leq 0 \quad (\text{Union req.})$$

and $X \geq 0, Y \geq 0$

- **Objective:**

Maximize the profit $P = 10X + 12Y$

- (Done! We just want to set the problem up!)

3.2: Example 2. Nutrition

- A Food-and-Nutrition-Science student was asked to design a diet for someone with iron and vitamin B deficiencies
- The student said the person should get at least 2400mg of iron, 2100mg of vitamin B_1 , and 1500mg of vitamin B_2 (over 90 days)
- The student recommended two brands of vitamins:

	Brand A	Brand B	Min. Req
Iron	40mg	10mg	2400mg
B_1	10mg	15mg	2100mg
B_2	5mg	15mg	1500mg
Cost:	\$0.06	\$0.08	

- The client asked the student to recommend the **cheapest** solution
- How many pills of each brand should the person get in order to meet the nutritional requirements at the minimal cost?

3.2: Example 2. Setting it up

- **Variables:**

X = number of pills of brand A

Y = number of pills of brand B

- **Constraints:**

$$40X + 10Y \geq 2400 \quad (\text{Iron})$$

$$10X + 15Y \geq 2100 \quad (\text{B1})$$

$$5X + 15Y \geq 1500 \quad (\text{B2})$$

and $X \geq 0, Y \geq 0$

- **Objective:**

Minimize cost $C = 0.06X + 0.08Y$

3.2: Example 3. Shipping costs

- You hit the big time, Mr. or Ms. Big Shot.
You've got two manufacturing plants and two assembly plants
- Your assembly plants A1 and A2 need 80 and 70 engines
- Your production plants can produce up to 100 and 110 engines
- The shipping costs are:

From	To assembly plant	
	A1	A2
P1	100	60
P2	120	70

- How many engines should each production plant ship to each assembly plant to meet the production goals at the minimum shipping cost?

3.2: Example 3. Setting it up (1/3)

- What do you have control over? Four things?

X = Number of engines from P1 to A1

Y = Number of engines from P1 to A2

Z = Number of engines from P2 to A1

ξ = Number of engines from P2 to A2

- But do we really need all these variables?

How many engines does A1 even want?

- $X + Z = 80$ and $Y + \xi = 70$

- Why not just use X and Y ?

Z and ξ are just “the rest”

3.2: Example 3. Setting it up (2/3)

- What are the requirements?
- Sanity is complicated: $X \geq 0$, $Y \geq 0$, $Z \geq 0$, $\xi \geq 0$
- But wait, we got rid of Z and ξ !
No big deal, just don't ship more than needed!
- Sanity: $0 \leq X \leq 80$ and $0 \leq Y \leq 70$
- Only other constraint is production capacity:
- $X + Y \leq 100$ from P1 capacity
- $Z + \xi \leq 110$ from P2 capacity
- Rewrite P2 as $(80 - X) + (70 - Y) \leq 110$ really just $40 \leq X + Y$

3.2: Example 3. Setting it up (3/3)

- What is the goal?
- Cost is complicated: $100X + 60Y + 120Z + 70\xi$
- Rewrite as $100X + 60Y + 120(80 - X) + 70(70 - Y)$
- Simplifies to $C = 9600 - 20X + 4900 - 10Y = 14500 - 20X - 10Y$
- Ok, but we need an executive summary, this was too long!

3.2: Example 3. Summary

- **Variables:**

X = Number of engines from P1 to A1

Y = Number of engines from P1 to A2

$80 - X$ = Number of engines from P2 to A1 (the rest of A1's demand)

$70 - Y$ = Number of engines from P2 to A2 (the rest of A2's demand)

- **Constraints:**

$$X + Y \leq 100 \quad (\text{P1 max production})$$

$$X + Y \geq 40 \quad (\text{P2 max production})$$

$$X \leq 80 \quad (\text{sanity, A1 max demand})$$

$$Y \leq 70 \quad (\text{sanity, A2 max demand})$$

and $X \geq 0, Y \geq 0$

- **Objective:**

minimize shipping cost $C = 14500 - 20X - 10Y$

3.2: Example 4. Fancy shipping

- Two plants P1 and P2 and three warehouses W1, W2, W3
- Shipping costs are in the following table:

	W1	W2	W3
P1	20	8	10
P2	12	22	18

- Maximum production and minimum requirements are:

	Prod.
P1	400
P2	600

	W1	W2	W3
Req	200	300	400

3.2: Example 4. Setting it up (1/3)

- We honestly have six variables! We'd run out of letters.
- $X_1, X_2, X_3, X_4, X_5, X_6$ are six different variables
- They are pronounced "Ecks One, Ecks Two, Ecks Three, ..."
- The number is just like a serial number, it doesn't mean multiply or square or anything like that
- So our variables are:
 - X_1 = number to ship from P1 to W1
 - X_2 = number to ship from P1 to W2
 - X_3 = number to ship from P1 to W3
 - X_4 = number to ship from P2 to W1
 - X_5 = number to ship from P2 to W2
 - X_6 = number to ship from P2 to W3

3.2: Example 4. Setting it up (2 and 3/3)

- What are the constraints?

Max production, and min reception

$$x_1 + x_2 + x_3 \leq 400 \quad (\text{P1 max prod})$$

$$x_4 + x_5 + x_6 \leq 600 \quad (\text{P2 max prod})$$

$$x_1 + x_4 \geq 200 \quad (\text{W1 min supply})$$

$$x_2 + x_5 \geq 300 \quad (\text{W2 min supply})$$

$$x_3 + x_6 \geq 400 \quad (\text{W3 min supply})$$

and $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_5 \geq 0$, and $x_6 \geq 0$.

- What is the objective?

Minimize cost: $C = 20x_1 + 8x_2 + 10x_3 + 12x_4 + 22x_5 + 18x_6$