MA162: Finite mathematics

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SCHEDULE:

- HW 3.1-3.3, 4.1 (Late)
- HW 2.5-2.6 due Friday, Mar 01, 2013
- Exam 2, Monday, Mar 04, 2013, from 5pm to 7pm
- HW 5.1 due Friday, Mar 08, 2013
- Spring Break, Mar 09-17, 2013
- HW 5.2-5.3 due Friday, Mar 22, 2013

Today we will cover 2.6: matrix inverse, and use it to make a spreadsheet solve LPPs

2.5: An easy multiplication, the identity

- There is a matrix that doesn't change things when it multiplies against them:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 11 & 12 & 13 & \dots \\ 21 & 22 & 23 & \dots \\ 31 & 32 & 33 & \dots \\ 41 & 42 & 43 & \dots \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 11 + 0 \cdot 21 + 0 \cdot 31 + 0 \cdot 41 & 12 & 13 & \dots \\ 0 \cdot 11 + 1 \cdot 21 + 0 \cdot 31 + 0 \cdot 41 & 22 & 23 & \dots \\ 0 \cdot 11 + 0 \cdot 21 + 1 \cdot 31 + 0 \cdot 41 & 32 & 33 & \dots \\ 0 \cdot 11 + 0 \cdot 21 + 0 \cdot 31 + 1 \cdot 41 & 42 & 43 & \dots \end{bmatrix}$$

- Make sure the size of the matices match though!

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$$=\begin{bmatrix}1\cdot 11+0\cdot 21+0\cdot 31+0\cdot 41 & 12 & 13 & \dots\\0\cdot 11+1\cdot 21+0\cdot 31+0\cdot 41 & 22 & 23 & \dots\\0\cdot 11+0\cdot 21+1\cdot 31+0\cdot 41 & 32 & 33 & \dots\\0\cdot 11+0\cdot 21+0\cdot 31+1\cdot 41 & 42 & 43 & \dots\end{bmatrix}$$

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2.5: Multiplication to solve one equation

- $\frac{2}{3} \cdot \frac{3}{2} = 1$
- To solve $\frac{3}{2}x = 9$ we just multiply by $\frac{2}{3}$
- $x = \frac{2}{3}9 = 6$
- We are multiplying both sides by $\frac{2}{3}$
- Left side turns out nice and boring:

$$\frac{2}{3} \cdot \left(\frac{3}{2}x\right) = \left(\frac{2}{3} \cdot \frac{3}{2}\right)x = (1)x = x$$

2.5: Multiplication to solve a system

- Matrix version:

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- To solve:

$$\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

- Just multiply:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} (1)(5) + (2)(7) \\ (1)(5) + (3)(7) \end{pmatrix} = \begin{pmatrix} 19 \\ 26 \end{pmatrix}$$

2.6: Matrix division

- There are several ways to do matrix division, see book for tricks
- We'll cover one systematic, basically easy way
- And we **already know it**, we just use RREF:
- If you know A and B, then to solve $AX = B$
  put the augmented matrix $(A|B)$ into RREF as $(I|X)$
- In other words, $RREF(A|B) = (I|X)$
- **inverses** are solving $AX = I$, $X = A^{-1}$, so we use RREF there too

2.6: Using RREF to solve the system

$$\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

- Make augmented matrix and RREF

$$\left(\begin{array}{rr|r} 3 & -2 & 5 \\ -1 & 1 & 7 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{rr|r} -1 & 1 & 7 \\ 3 & -2 & 5 \end{array}\right) \xrightarrow{R_2+3R_1}$$

$$\left(\begin{array}{rr|r} -1 & 1 & 7 \\ 0 & 1 & 26 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{rr|r} -1 & 0 & -19 \\ 0 & 1 & 26 \end{array}\right) \xrightarrow{-R_1} \left(\begin{array}{rr|r} 1 & 0 & 19 \\ 0 & 1 & 26 \end{array}\right)$$

- Find inverse is almost exactly the same

$$\left(\begin{array}{rr|rr} 3 & -2 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{rr|rr} -1 & 1 & 0 & 1 \\ 3 & -2 & 1 & 0 \end{array}\right) \xrightarrow{R_2+3R_1}$$

$$\left(\begin{array}{rr|rr} -1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{rr|rr} -1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow{-R_1} \left(\begin{array}{rr|rr} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right)$$

2.6: Why is the inverse useful?

- The inverse allows you to solve $AX = B$ using matrix multiplication instead of RREF
- $A^{-1}A = I$
- $A^{-1}AX = IX = X$
- If $AX = B$, then multiply both sides on the left by A^{-1}
  then $A^{-1}AX = A^{-1}B$
  so $X = A^{-1}B$
- Multiply by the inverse does the same thing as the long RREF
- Of course to find the inverse, we use RREF

4.1: Using the inverse to calculate RREF

- If a square matrix has an inverse, its RREF is the identity, very dull
- To find the RREF of a non-square matrix, we multiply by the inverse of a submatrix
- If the matrix is wide, then we choose one column per row to be in the submatrix
- We multiply the original by the inverse of the submatrix
- Those columns form the submatrix become pivots (one 1 and the rest 0s)
- The others change so that the X,Y,Z solutions don't change

4.1: Using the inverse to calculate RREF

- A simple way to do the simplex algorithm in google docs
- The submatrix is specified by putting 0 or 1 below the column

  (0 is FREE, 1 is PIVOT)
- The formula for the submatrix is `FILTER( Matrix, PivotList )`
- The formula for the RREF is:

  `MMULT(MINVERSE(FILTER(Matrix,PivotList)),Matrix)`
- See this google doc for an example from the practice exam.

  It also calculates the ratios to help find pivot rows