# MA162: Finite mathematics

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March 6th, 2013

SCHEDULE:

- HW 2.5-2.6, 3.1-3.3, 4.1 (Late)
- HW 5.1 due Friday, Mar 08, 2013
- Spring Break, Mar 09-17, 2013
- HW 5.2-5.3 due Friday, Mar 22, 2013
- HW 6A due Friday, Mar 29, 2013

Today we will cover the time value of money.

# Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations

# 5.1: Interest

- Businesses often need short-term use of expensive assets,
  so find renting attractive (often tax-deductible)
- Sometimes what a business needs most is just cash.
  In a small business, you don't make money every day.
  A successful small business does make money,
  so can repay the money in the future.
- How can they rent cash?
  Why would somebody give them money today?
  For the promise of more money in the future. **Interest**
- How much more?
  - The more money being loaned, the more interest. **Principal**
  - The longer the money is loaned, the more interest. **Time**

# 5.1: Simple interest

- For short term loans, people use a **simple** model for interest

$$I = Prt$$

- There is the **Principal**, the amount of money borrowed, like \$100
- There is a **rate** of interest, like 10% per year
- There is a **time** period, after which the money is due, like 1 year
- There is the **Interest**, the extra money due at the end,

  like $(\$100) \cdot (10\% \text{ per year}) \cdot (1 \text{ year}) = \$10$.

# 5.1: Simple interest examples

$$I = Prt$$

- If \$100 is lent at 10% interest per year for six months, then

$$I = (\$100) \cdot (10\% \text{ per year}) \cdot (\tfrac{1}{2} \text{ year}) = \$5$$

- If \$100 is lent at 7% interest per year for three months, then

$$I = (\$100) \cdot (7\% \text{ per year}) \cdot (\tfrac{1}{4} \text{ year}) = \$1.75$$

- If \$325 is lent at 12% interest per year for five months, then

$$I = (\$325) \cdot (12\% \text{ per year}) \cdot (\tfrac{5}{12} \text{ year}) = \$16.25$$

# 5.1: Consumer example

- My Brother-in-Law's electricity bill came too soon one month
- Bill was \$46.40 now, but \$48.72 if 3 days late
- He didn't have the money now, but would have it in a week (IRS refund)
- He did have a 48% APR credit card carrying a balance (4% interest per month)
- A pawn shop would loan him the money for one month
  2% interest per month, \$5 fee
- Which is cheaper:
  (L) Pay it late
  (R) Put it on the credit card, and pay the credit card
  (B) Pawn his watch for a month, then pay it back

# 5.1: Let's just see how much each costs

- (L) is easy: \$48.72 total, \$2.32 in interest
- (R) is easy: \$46.40 plus 4% $=$ \$46.40(1.04) $=$ \$48.26
- (B) is easy: \$46.40 plus 2% plus \$5 $=$ \$46.40(1.02) $+ 5 =$ \$52.33
- Decision is also easy: credit card is the cheapest
- If he had the money now, then cheapest was to pay it now \$46.60
- There is a price to not having money

# 5.1: More examples

- What is the simple (yearly) interest rate if \$100 is loaned for 3 months with \$5 interest due?

$$\begin{aligned} P &= \$100 \\ t &= 1/4 \text{ year} \\ I &= \$5 \\ r &= ? \end{aligned} \qquad \begin{aligned} I &= Prt \\ 5 &= (100)(r)(1/4) \\ 5 &= 25r \\ r &= 1/5 = 20\% \end{aligned}$$

- If the interest rate is 7% and \$9.10 interest is due after three months, how much was loaned?

$$\begin{aligned} r &= 7\% \text{ per year} \\ t &= 1/4 \text{ year} \\ I &= \$9.10 \\ P &= ? \end{aligned} \qquad \begin{aligned} I &= Prt \\ \$9.1 &= P(7\%)(1/4) \\ \$36.4 &= P(7\%) \\ P &= \$36.4/7\% = \$520 \end{aligned}$$

# 5.1: Reinvesting interest

- If you ran a bank, would you offer simple interest or compound interest?
- Would you give your customers interest on their interest?
- Maybe you think you'd save money by not giving interest on interest,
- But then the customer would just withdraw the interest
- Invest it elsewhere
- A bank makes money by having money.
- Simple interest is only used for short or fixed term loans

# 5.1: Compound interest

- The most basic formula for compound interest is:

$$A = P(1+i)^n$$

- the **Principal** is the amount initially borrowed, like \$100
- the **interest rate** per compounding period, like 10% per month
- the **number** of compounding periods that have passed, like 2 months
- the **Accumulated Amount** of money due, both the principal and the interest, like

$$(\$100)(1+10\%)^2 = (\$100)(1.10)^2 = \$121$$

# 5.1: Compound interest examples

$$A = P(1+i)^n$$

- If you borrow \$100 at 10% per month, compounded monthly, for six months you owe

$$(\$100) \cdot (1.1)^6 \approx \$177.16$$

- If you borrow \$100 at 10% per month, compounded monthly, for nine months you owe

$$(\$100) \cdot (1.1)^9 \approx \$235.79$$

- If you borrow \$100 at 10% per month, compounded monthly, for twelve months you owe

$$(\$100) \cdot (1.1)^{12} \approx \$313.84$$

# 5.1: Why does the formula work?

- If you borrow \$100 at 10% per month, compounded monthly, for one month you owe

$$\$100 + (\$100)(10\%) = (\$100) \cdot (1 + 10\%) = (\$100) \cdot (1.1) = \$110$$

- If you borrow it for another month, you owe

$$\$110 + (\$110)(10\%) = (\$110)(1 + 10\%) = (\$110)(1.1)$$

$$= (\$100)(1.1)(1.1) = (\$100)(1.1)^2 = \$121$$

- If you borrow it for another month, you owe

$$\$121 + (\$121)(10\%) = (\$121)(1 + 10\%) = (\$121)(1.1)$$

$$= (\$100)(1.1)^2(1.1) = (\$100)(1.1)^3 = \$133.10$$

- Now $\xrightarrow{\times 1.10}$ One Month in the Future $\xrightarrow{\times 1.10}$ Two months in the future

# 5.1: Confusing customers for fun and profit (APR)

- Stating interests rates in terms of months, fortnights, or furlongs makes it hard to compare interest rates
- A simple way to handle this is to multiply the rate by how many periods there are per year, to "convert" to a yearly rate, like $(10\% \text{ per month}) \cdot (12 \text{ months per year}) = 120\% \text{ per year}$
- The **nominal rate** is this rate, "120% interest per year, compounded monthly"
- To convert from a nominal rate to a per-period rate just divide by the number of periods
- a nominal rate of 12% per year compounded monthly is a rate of $(12\% \text{ per year})/(12 \text{ months per year}) = 1\%$ per month

# 5.1: Effective interest rate

- In the U.S. the 1968 Truth in Lending Act required lenders to advertise the **effective** annual percentage rate
- The true calculation is complicated, depends on your jurisdiction, and takes into account certain fees and penalties.
- In MA162, the formula is not so complicated. You just calculate the interest for one year.
- For instance, a nominal APR of 120% compounded monthly results in

$$(1+\frac{1.20}{12})^{12}-1 = (1+0.10)^{12}-1 = 1.1^{12}-1 \approx 2.13843 = 213.843\%$$

- In general

$$r_{eff} = (1 + \frac{r}{m})^m - 1$$

# 5.1: Summary

- Today we learned **simple interest**, **compound interest**, and the **effective interest rate**.
- We used the words **interest**, **principal**, **interest rate**, **compounding period**, **nominal rate**, **accumulated amount**.
- You are now ready to complete HW 5.1
- Make sure to take advantage of office hours, and have your questions ready for your next recitation