MA162: Finite mathematics

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March 25th, 2013

SCHEDULE:

- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3 (Late)
- HW 6A due Friday, Mar 29, 2013
- HW 6B-6C due Friday, Apr 5, 2013
- Exam 3, Monday, Apr 8, 2013
- HW 7A due Friday, Apr 12, 2013

Today we cover 6.1 (sets) and the pigeonhole principle

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations

6.1: Life before sets

- We are going to be doing some hard counting problems.
- To make it easier, we need to be able to talk about the things we are counting.
- When we counted money, or hours of labor, or short sleeve shirts, we had variables to denote the number. $x = 5$ hours, or $y = 10$ shirts.
- If you had \$5 in one bank account and \$10 in another, you had \$5+\$10 = \$15 total. The numbers were all that mattered.
- Unfortunately life rarely divides nicely into separate accounts, and numbers cannot describe many of these aspects.

6.1: More than numbers can say

- We are going to be counting more complicated things now.
- If your friend Jimmy says you can borrow their car Monday, Tuesday, and Wednesday, then that is 3 days you've got a car.
- If your friend Timmy says you can borrow their car Tuesday, Thursday, and Friday, then that is 3 days you've got a car.
- How many days total can you borrow a car?

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- If your friend Timmy says you can borrow their car Tuesday, Thursday, and Friday, then that is 3 days you've got a car.
- How many days total can you borrow a car?
- Well, Monday, Tuesday, Wednesday, Thursday, Friday is five days.
- But $5 \neq 3 + 3$. Numbers are not enough.

6.1: Sets to name the things we are counting

- If we let J be the days Jimmy lets us have the car, then

$$J = \{ \text{ Monday, Tuesday, Wednesday } \}$$

- If we let T be the days Timmy lets us have the car, then

$$T = \{ \text{ Tuesday, Thursday, Friday } \}$$

- The days when at least one of them let us use the car is the **union** of the two sets

$$J \cup T = \{ \text{ Monday, Tuesday, Wednesday, Thursday, Friday } \}$$

- The days when both of them let use the car is the **intersection** of the two sets

$$J \cap T = \{ \text{ Tuesday } \}$$

6.1: More sets

- We can have sets of numbers $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then:
- $A \cup B = \{1, 2, 3, 4, 5\}$
- $A \cap B = \{3\}$
- $A - B = \{1, 2\}$ is the difference, the things in A that are not in B
- We can write down sets in funny ways:
  $A = \{3, 2, 1\} = \{1, 1, 1, 1, 1, 2, 2, 3\}$
- We can describe them in words, "A is the set of positive integers whose square is a one digit number."

6.1: Equality drill

- Two sets are **equal** if they have the same elements.
- $\{1, 2, 3\} \stackrel{?}{=} \{1, 2, 3\}$
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- $\{1, 2, 3\} \stackrel{?}{=} \{1, 2, 2, 3, 3, 3\}$
- $\{1, 2, 3\} \stackrel{?}{=} \{$ positive integers whose square has one digit $\}$
- $\{1, 2, 3\} \stackrel{?}{=} \{$ odd numbers less than 4 $\}$

6.1: Equality drill

- Two sets are **equal** if they have the same elements.
- $\{1, 2, 3\} = \{1, 2, 3\}$
  Yes! Exactly the same.
- $\{1, 2, 3\} \neq \{1, 2\}$
  No! Right hand set is missing "3"
- $\{1, 2, 3\} = \{3, 1, 2\}$
  Yes! Order does not matter.
- $\{1, 2, 3\} = \{1, 2, 2, 3, 3, 3\}$
  Yes! Repeats don't matter.
- $\{1, 2, 3\} = \{$ positive integers whose square has one digit $\}$
  Yes! Long-winded doesn't matter.
- $\{1, 2, 3\} \neq \{$ odd numbers less than 4 $\}$
  No! Right hand set is missing "2"

6.1: Union and intersection drill

- $\bigcup$ The **union** includes anything in either, and is big. $\bigcup$
- $\bigcap$ The **intersection** includes only those in both, and is small. $\bigcap$
- $\{1, 2, 3\} \cup \{3, 4, 5\} =$
- $\{1, 2, 3\} \cap \{3, 4, 5\} =$
- $\{1, 2, 3\} \cup \{1\} =$
- $\{1, 2, 3\} \cap \{1\} =$

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- $\{1,2,3\} \cap \{3,4,5\} = \{3\}$
- $\{1,2,3\} \cup \{1\} = \{1,2,3\}$
- $\{1,2,3\} \cap \{1\} = \{1\}$

6.1: Difference drill

- $-$ The **difference** keeps the first, but not in the second. $-$
- $\{1, 2, 3\} - \{1\} =$
- $\{1, 2, 3\} - \{2, 3\} =$
- $\{1, 2, 3\} - \{3, 4, 5\} =$
- $\{1, 2, 3\} - \{4, 5, 6\} =$
- $\{1, 2, 3\} - \{1, 2, 3\} =$

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- $\{1,2,3\} - \{2,3\} = \{1\}$
- $\{1,2,3\} - \{3,4,5\} = \{1,2\}$
- $\{1,2,3\} - \{4,5,6\} = \{1,2,3\}$
- $\{1,2,3\} - \{1,2,3\} = \{\}$ The **empty set** containing nothing.

6.1: Laws of sets

- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$, but what about $\{3, 4, 5\} \cup \{1, 2, 3\}$?

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- Order of union does not matter

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- What about $\{1,2,3\} \cap \{3,4,5\}$ versus $\{3,4,5\} \cap \{1,2,3\}$?

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- Both are $\{3\}$.
- $A = \{1,2,3\}$, and $B = \{3,4,5\}$. Compare $A \cap B$ and $A - B$.

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- $A \cap B = \{3\}$ and $A - B = \{1,2\}$

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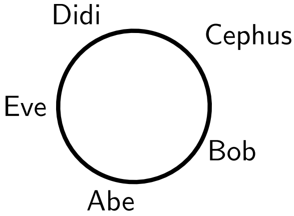
- Order of union does not matter
- What about $\{1, 2, 3\} \cap \{3, 4, 5\}$ versus $\{3, 4, 5\} \cap \{1, 2, 3\}$?
- Both are $\{3\}$.
- $A = \{1, 2, 3\}$, and $B = \{3, 4, 5\}$. Compare $A \cap B$ and $A - B$.
- $A \cap B = \{3\}$ and $A - B = \{1, 2\}$
- $A = (A \cap B) \cup (A - B)$

6.1: Counting cards

- Five friends are playing cards with a standard 52 card deck



- The deck has the following cards:
A♡ 2♡ 3♡ 4♡ 5♡ 6♡ 7♡ 8♡ 9♡ 10♡ J♡ Q♡ K♡
A♢ 2♢ 3♢ 4♢ 5♢ 6♢ 7♢ 8♢ 9♢ 10♢ J♢ Q♢ K♢
A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣
A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠
- 50 cards have been dealt out, 10 to each person
- Why must one of the suits ($\heartsuit, \diamondsuit, \clubsuit, \spadesuit$) be completely dealt out?

6.1: Some answers

- There are 52 cards, but 50 have been dealt out, leaving two undealt. If no suit was dealt out completely, then each suit is missing one card. That is four cards missing, but only two cards undealt. Impossible.

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	$\heartsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$	Total
A	$A_\heartsuit$	$A_\diamondsuit$	$A_\clubsuit$	$A_\spadesuit$	$= 10$
B	$B_\heartsuit$	$B_\diamondsuit$	$B_\clubsuit$	$B_\spadesuit$	$= 10$
C	$C_\heartsuit$	$C_\diamondsuit$	$C_\clubsuit$	$C_\spadesuit$	$= 10$
D	$D_\heartsuit$	$D_\diamondsuit$	$D_\clubsuit$	$D_\spadesuit$	$= 10$
E	$E_\heartsuit$	$E_\diamondsuit$	$E_\clubsuit$	$E_\spadesuit$	$= 10$
Total	≤ 12	≤ 12	≤ 12	≤ 12	$\leq {}_{48} \backslash {}^{=50}$

- In other words, if we avoid finishing suits, we only deal 48 cards, not 50.

6.1: Counting

- Five friends are playing cards with a standard 52 card deck
- 50 cards have been dealt out, 10 each
- (b) At least two people have at least one club ♣
- (c) At least one person has at least two clubs ♣
- (d) Every player has at least 3 of the same suit
- (e) Some pair of neighbors has at least 6 of the same suit (combined)

6.1: Some answers

- (b) At least 11 clubs dealt. No one person can get them all, so at least two people.

- (c) At least 11 clubs dealt. If nobody has even two clubs, that is only 5 clubs dealt.

- (d) Otherwise, each player has at most two of each suit, only eight cards each, not ten

- (e) Similar to part (a):

$$A_\heartsuit + B_\heartsuit \leq 5,\ A_\diamondsuit + B_\diamondsuit \leq 5,\ A_\clubsuit + B_\clubsuit \leq 5,\ A_\spadesuit + B_\spadesuit \leq 5$$

$$B_\heartsuit + C_\heartsuit \leq 5,\ B_\diamondsuit + C_\diamondsuit \leq 5,\ B_\clubsuit + C_\clubsuit \leq 5,\ B_\spadesuit + C_\spadesuit \leq 5$$

$$C_\heartsuit + D_\heartsuit \leq 5,\ C_\diamondsuit + D_\diamondsuit \leq 5,\ C_\clubsuit + D_\clubsuit \leq 5,\ C_\spadesuit + D_\spadesuit \leq 5$$

$$D_\heartsuit + E_\heartsuit \leq 5,\ D_\diamondsuit + E_\diamondsuit \leq 5,\ D_\clubsuit + E_\clubsuit \leq 5,\ D_\spadesuit + E_\spadesuit \leq 5$$

$$E_\heartsuit + A_\heartsuit \leq 5,\ E_\diamondsuit + A_\diamondsuit \leq 5,\ E_\clubsuit + A_\clubsuit \leq 5,\ E_\spadesuit + A_\spadesuit \leq 5$$

Add up the columns, get $2T_\heartsuit \leq 25$, $2T_\diamondsuit \leq 25$, $2T_\clubsuit \leq 25$, $2T_\spadesuit \leq 25$, but one suit has $T = 13$, oops.

6.1: Summary

- Today we learned about **sets**, **union**, **intersection**, and **difference**.
- You are now ready to complete 6A. Better try 6B now.
- Make sure to take advantage of office hours