MA162: Finite mathematics

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University of Kentucky

March 27th, 2013

Schedule:

- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3 (Late)
- HW 6A due Friday, Mar 29, 2013
- HW 6B-6C due Friday, Apr 5, 2013
- Exam 3, Monday, Apr 8, 2013
- HW 7A due Friday, Apr 12, 2013

Today we cover 6.2 (counting unions) and the pigeonhole principle

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
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  - Multiplication principle
  - Permutations and combinations

6.2: Counting the missing piece

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- Well, it is hard to say, right?
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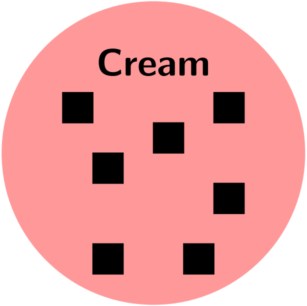

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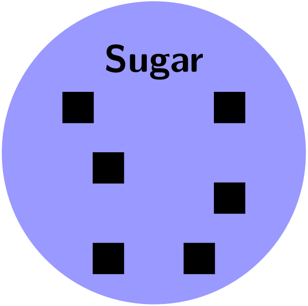

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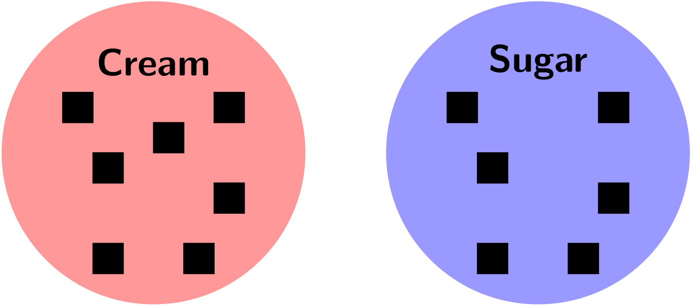

- $60 + 70 = 130$ is way too big. What happened?

6.2: The overlap

- In order to figure out how many take it black, we need to know how many take it with cream or sugar or both.

$$\#\text{Black} = 100 - n(C \cup S)$$

- However, in order to find out how many take either, we kind of need to know how many take both:

$$n(C \cup S) = n(C) + n(S) - n(C \cap S) = 70 + 60 - n(C \cap S)$$

- So what if 50 people took both?

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- Then $n(C \cup S) = 130 - 50 = 80$ and so $100 - 80 = 20$ took neither.

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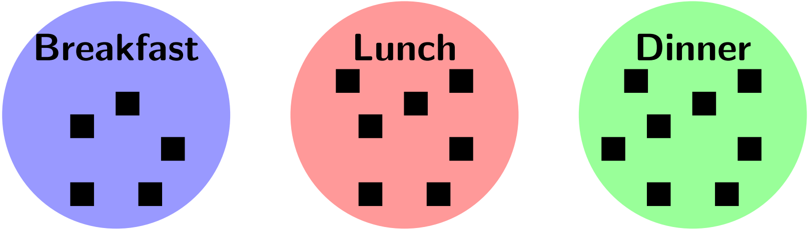
- What if those were exactly the 20 people that didn't eat dinner?
- Could be 0%, could be 50%. We need to know more!

6.2: More information and a picture

- If we let B, L, D be the sets of people, then we are given

$$n(B) = 50, n(L) = 70, n(D) = 80,$$

and we want to know $n(B \cap L \cap D)$.



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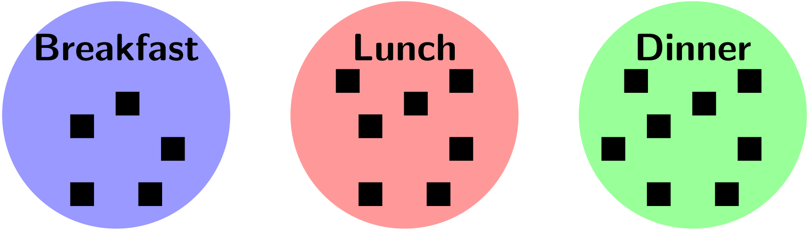

- What if we find out:

$$n(B \cap L) = 30, n(B \cap D) = 40, n(L \cap D) = 40$$

We can find the overlaps!

6.2: More information and a formula

- Just like before, there is a formula relating all of these things:

  $$n(B \cup L \cup D) = n(B) + n(L) + n(D) - n(B \cap L) - n(L \cap D) - n(D \cap B) + n(B \cap L \cap D)$$

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- We plugin to get:

$$100 = 50 + 70 + 80 - 30 - 40 - 40 + n(B \cap L \cap D)$$

$$100 = 200 - 110 + n(B \cap L \cap D)$$

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- **Inclusion-exclusion formula** will be given on the exam, but make sure you know how to use it!

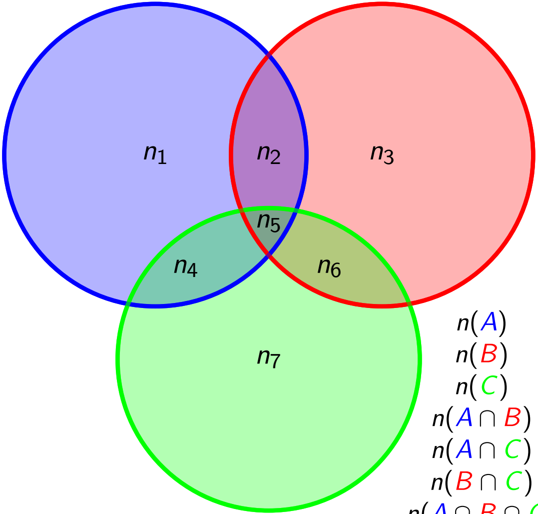
6.2: Old exam question

- A survey of 100 students asked for their opinions about pizza. They were specifically whether they liked pepperoni, mushrooms, and garlic.
  - 43 students liked pepperoni.
  - 39 students liked mushrooms.
  - 40 students liked garlic.
  - 12 students liked both pepperoni and mushrooms.
  - 14 students liked both pepperoni and garlic.
  - 13 students liked both mushrooms and garlic.
  - 9 students liked all three toppings.
- How many students surveyed did not like any of the three toppings?
- How many students surveyed liked at least two of the toppings?

6.2: Old exam question

- A, B and C are sets with 64, 57, and 58 members respectively.
- If $A \bigcup B$ has 82 members,
  then $A \bigcap B$ has ______________members.
- If $A \bigcap C$ has 35 members,
  then $A \bigcup C$ has ______________members.
- If $B - C$ has 25 members,
  then $B \bigcap C$ has ______________members.
- If $A \cap B \cap C$ has 20 members (and all the previous is true),
  then the union of these three sets has ______________members.

6.2: Picture and formula



$$
\begin{aligned}
n(A) &= n_1 + n_2 + n_4 + n_5 \\
n(B) &= n_2 + n_3 + n_5 + n_6 \\
n(C) &= n_4 + n_5 + n_6 + n_7 \\
n(A \cap B) &= n_2 + n_5 \\
n(A \cap C) &= n_4 + n_5 \\
n(B \cap C) &= n_5 + n_6 \\
n(A \cap B \cap C) &= n_5 \\
n(A \cup B \cup C) &= n_1 + n_2 + n_3 + n_4 \\
&\quad n_5 + n_6 + n_7
\end{aligned}
$$

6.2: Summary

- We learned the notation $n(A) =$ the number of things in the set A

- We learned the basic inclusion-exclusion formulas:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

and

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

- Make sure to complete HW 6.2 and read over the old exam questions

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- If 10 people (out of however many) have their test come back positive, about how many are users?

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- Suppose we know that there were 200 people in the testing pool. About how many were drug users?
- Assuming exactly 5% of non-users returned positive, there is a unique answer. Let me know when you've found it.

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- All 10 are false positives; 100% wrong, but 95% accurate? Be careful what you are counting.