MA162: Finite mathematics

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University of Kentucky

April 1st, 2013

SCHEDULE:

- Written Project due today. Please turn it in at the end of class.
- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3, 6A (Late)
- HW 6B-6C due Friday, Apr 5, 2013
- Exam 3, Monday, Apr 8, 2013
- HW 7A due Friday, Apr 12, 2013
- HW 7B due Friday, Apr 19, 2013

Today we cover 6.3 (multiplication principle)

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
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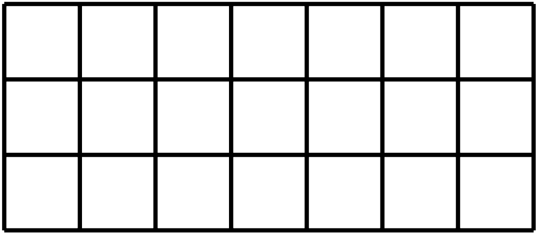
  - Multiplication principle
  - Permutations and combinations

6.3: What is multiplication?

- How many small 1×1 squares in this figure?



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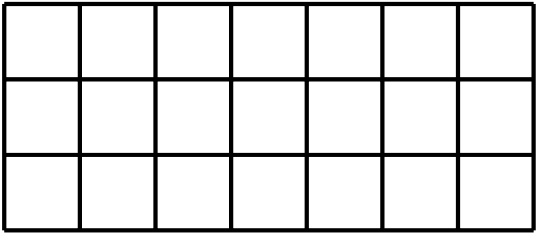

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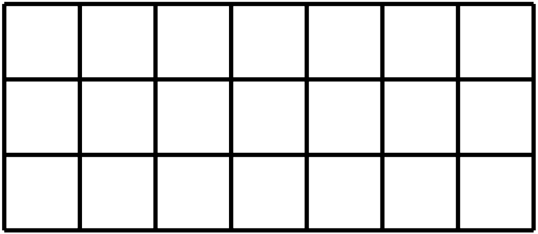

- Each column has 3 squares, there are 7 columns, so $3 \cdot 7 = 21$
- Counting each square is slower and error-prone.

6.3: Three square meals a day

- You decide to brush your teeth after every meal, but are worried about the toothpaste consumption. You use about 1% of the tube every time you brush. How many weeks will it last?

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- How many brushes per week?

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Lun							
Din							

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- So 21 brushes per week; takes less than 5 weeks to use up a tube.

6.3: A rainbow of possibilities

- You are working on a dazzling fashion project and have seven dyes: **Red**, **Orange**, **Yellow**, **Green**, **Blue**, **Indigo**, and **Violet**. You've got three types of fabric: Burlap, Cotton, and Denim.

  How many different color/texture combinations do you have?

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- Again $(3)(7) = 21$

	Red	**Ora**	**Yel**	**Gre**	**Blu**	**Ind**	**Vio**
Bur							
Cot							
Den							

6.3: Multiplication principle

- If you have two independent choices (color and fabric)
- The total number of choices is the product of the number of individual choices
- You get to choose one of those and one of these
- If you have three independent choices, you multiply all three counts

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  (12)(2)(2)=48

6.3: Dice

- If you roll a white die (⚀, ⚁, ⚂, ⚃, ⚄, ⚅)

  and a blue die (⚀, ⚁, ⚂, ⚃, ⚄, ⚅),

  how many possible outcomes are there?

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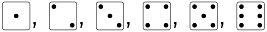
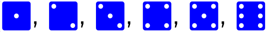
- If you roll a white die ()

  and a blue die (),

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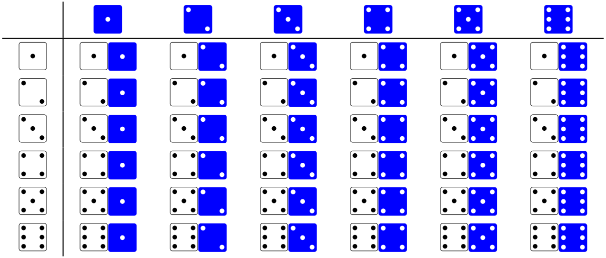
- A picture is easier:



36 ways

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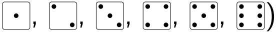
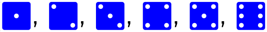
- If you roll a white die ()

  and a blue die (),

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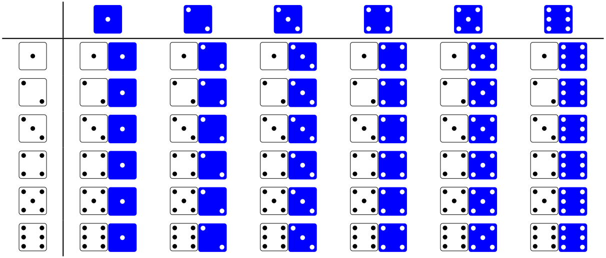

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- How many ways to get an odd number?

6.3: Flipping coins

- Get a **penny**, a **nickel**, and a **dime**. Flip all three.

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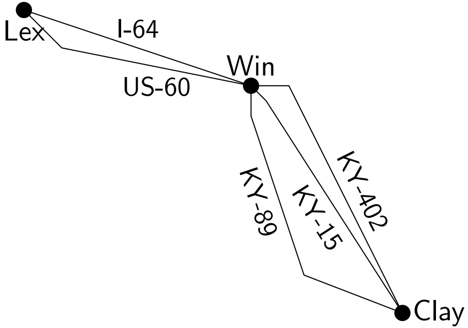
- How many ways to get more heads than tails?

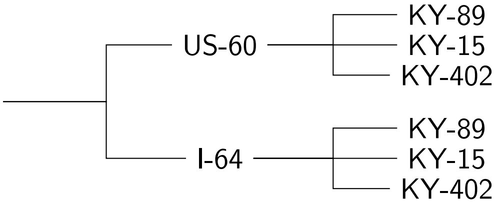
6.3: Drawing the possibilities

- There are two main ways to get to Winchester from Lexington: Winchester Rd (US-60) and I-64.
  From Winchester, there are three main ways to Clay City: KY-89, KY-15, and the Mountain Parkway (KY-402).
  How many different ways are there from Lexington to Clay City using these routes?



6.3: Trees for counting

- We can unfold the map to make the possibilities clearer:



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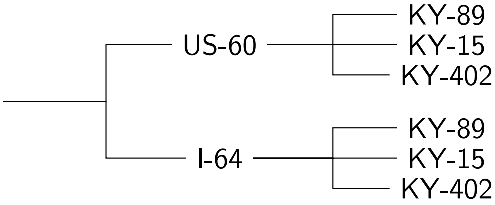

- This is a decision tree. Note how the decision to be made after I-64 is the same as the decision to be made after US-60. The first choice does not affect the second choice. The choices are **independent**.

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- How many cars are in Kentucky?
- 4 million people, about 4 million vehicles, 2 million of which probably have standard plates

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- $(5)(10)(6) \cdot (4)(9)(5) = 54000$.

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- Six possibilities for first (H,O,R,S,E,Y)
  and five for second (the remaining five)