MA162: Finite mathematics

Jack Schmidt

University of Kentucky

April 3rd, 2013

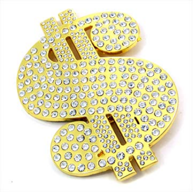
Schedule:

- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3, 6A (Late)
- HW 6B-6C due Friday, Apr 5, 2013
- Exam 3, Monday, Apr 8, 2013
- HW 7A due Friday, Apr 12, 2013
- HW 7B due Friday, Apr 19, 2013
- HW 7C due Friday, Apr 26, 2013

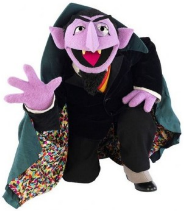
Today we cover 6.4 (combinations and permutations)

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans



- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations



6.4: Trifecta!

- Some people bet on horse races, a “Trifecta” bet is common
- You predict the first, second, and third place winners, in order.
- There are 14 contenders: **A**ccounting We Will Go, **B**usiness Planner, **C**orporate Finance, **D**ebt Sealing, **E**conomy Model, **F**iscal Filly, **G**ross Domestic Pony, **H**orse Resources, **I**nitial Pony Offering, **J**ust Another Horsey, **K**arpay Deeum, **L**OL Street, **M**arkety Mark, and **N**o Chance Vance
- Which ones will you choose? **A, B, C** or **L, N, E**?
- How many possibilities?

6.4: Counting the possibilities

______	______	______
1ST	2ND	3RD

- There are three places

6.4: Counting the possibilities

14		
1st	2nd	3rd

- There are three places
- There are 14 possibilities for first place,

6.4: Counting the possibilities

14	13	
1st	2nd	3rd

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place

6.4: Counting the possibilities

14	13	12
1ST	2ND	3RD

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place
- and only 12 left for third place

6.4: Counting the possibilities

$$\underset{\text{1ST}}{\underline{\quad 14 \quad}} \quad \underset{\text{2ND}}{\underline{\quad 13 \quad}} \quad \underset{\text{3RD}}{\underline{\quad 12 \quad}} = 2184$$

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place
- and only 12 left for third place
- That is $(14)(13)(12) = 2184$ total possibilities

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- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place
- and only 12 left for third place
- That is $(14)(13)(12) = 2184$ total possibilities
- If you bet 1000 times, only a 1 in 3 chance of winning at least once

6.4: Club officers

- The Variety Club has a President, a Vice President, a Secretary, and a Treasurer
- The V.C. has 6 members: Art, Ben, Cin, Dan, Eve, and Fin.
- But every day they want to assign a different set of officers
- Can they make it a year without exactly repeating the officer assignments?
- So maybe ABCD, then ABCE, then ABCF, then ABDC, then . . .

6.4: Counting the assignments

$\underline{\qquad\qquad}$ $\underline{\qquad\qquad}$ $\underline{\qquad\qquad}$ $\underline{\qquad\qquad}$

Pres *Vice* *Sec.* *Trs.*

- There are four positions, and order matters

6.4: Counting the assignments

6			
Pres	*Vice*	*Sec.*	*Trs.*

- There are four positions, and order matters
- There are 6 people available to president each day

6.4: Counting the assignments

6	5		
Pres	*Vice*	*Sec.*	*Trs.*

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP

6.4: Counting the assignments

6	5	4	
Pres	*Vice*	*Sec.*	*Trs.*

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary

6.4: Counting the assignments

6	5	4	3
Pres	*Vice*	*Sec.*	*Trs.*

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer

6.4: Counting the assignments

$$\frac{6}{Pres} \quad \frac{5}{Vice} \quad \frac{4}{Sec.} \quad \frac{3}{Trs.} = 360$$

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer
- There are $(6)(5)(4)(3) = 360$ possible assignments

6.4: Counting the assignments

$$\underset{Pres}{\underline{\quad 6 \quad}} \quad \underset{Vice}{\underline{\quad 5 \quad}} \quad \underset{Sec.}{\underline{\quad 4 \quad}} \quad \underset{Trs.}{\underline{\quad 3 \quad}} = 360$$

- There are four positions, and order matters
- There are 6 people available to president each day
- There are 5 people left to be VP
- There are 4 people left to be Secretary
- There are 3 people left to be Treasurer
- There are $(6)(5)(4)(3) = 360$ possible assignments
- Not enough for a calendar year, but certainly for a school year!

6.4: Always down by one?

- Do you always drop the number one?

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  How many ways can a P and a VP be chosen
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- There are two positions:

$$\underset{Pres}{\underline{\hspace{2cm}}} \quad \underset{Trs.}{\underline{\hspace{2cm}}}$$

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$$\underset{Pres}{\underline{\quad 10 \quad}} \qquad \underset{Trs.}{\underline{\qquad\quad}}$$

- There are ten people eligible for president

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$$\underset{Pres}{\underline{\quad 10 \quad}} \quad \underset{Trs.}{\underline{\quad 5 \quad}}$$

- There are ten people eligible for president
- But only five people left for vice president

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- Five boys and five girls are in a club.
  How many ways can a P and a VP be chosen
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$$\underset{Pres}{\underline{\quad 10 \quad}} \quad \underset{Trs.}{\underline{\quad 5 \quad}} = 50$$

- There are ten people eligible for president
- But only five people left for vice president
- That is $(5)(10) = 50$ different officer assignments

6.4: Spider shoes

- Spiders don't like to be put in boxes
- They will not conform to traditional notions of fashion
- This spider has 8 feet and 20 pairs of shoes
- How many ways can he wear 16 shoes (left on the left feet, and rights on the right)?
- How many ways without wearing any matching shoes?

6.4: Spelling

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- 7 choices for first, 6 for second, . . . , (7)(6)(5)(4)(3)(2)(1)

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- How many ways can one rearrange the letters of GLACIER?
- 7 choices for first, 6 for second, ..., $(7)(6)(5)(4)(3)(2)(1)$
- Shortcut name for this is $7!$, the **factorial** of 7

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- Well, a little different since there are two Ks

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- How many ways can one rearrange the letters of KENTUCKY?
- Well, a little different since there are two Ks
- 8! ways if we keep track of which K is which, then divide by two since each word like KENTUCKY appears twice as kENTUCKY and KENTUCkY.

$$8!/2 = 20160$$

6.4: Team players

- If there are 15 able bodied players, and we need to choose 11 of them to be on the field. We want four forwards, three midfielders, three defenders, and one goalie. We let the players themselves dynamically decide on the left/right/center. How many selections are possible?

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- (15)(14)(13)(12) choices of forwards counting order, but (4)(3)(2)(1) ways of re-ordering them, so $(15)(14)(13)(12)/((4)(3)(2)(1)) = 15!/(11!4!) = 1365$ ways ignoring order

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- Then $8!/(5!3!) = 56$ ways of choosing defenders ignoring order
- Then 5 ways of choosing the goalie.
- Total is: (1365)(165)(56)(5) ways of choosing the first string

6.4: Partial arrangements with repeats

- How many ways to arrange two letters from HIPPOPOTAMUS
- NOT (12)(11). Like with KENTUCKY (and KK vs KK) this counts PP and PP as different.
- NOT (12)(11)/2. Not every word is counted twice: HI is only counted once
- NOT (9)(8). That only covers the non-repeated ones.
- NOT (2)(1). That only covers the repeated ones.
- Oh, so it is $(9)(8)+(2)(1)=74$.

6.4: Larger partial arrangements

- How many ways to arrange three letters from HIPPOPOTAMUS?
- No-repeats (9)(8)7)
- or triples $(1)(1)(1)$, only PPP
- or one of these types of doubles: PP?, P?P, ?PP, OO?, O?O, ?OO; each one has 8 possibilities for the ?
- so $(9)(8)(7) + 1 + 8{+}8{+}8{+}8{+}8{+}8 = 553$
- Four letters is 3734, but too complicated for the exam