MA162: Finite mathematics

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April 10th, 2013

SCHEDULE:

- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3, 6A-6C (Late)
- HW 7A due Friday, Apr 12, 2013
- HW 7B due Friday, Apr 19, 2013
- HW 7C due Friday, Apr 26, 2013
- Final Exam Tuesday, Apr 30, 2013 from 6pm to 8pm (new rooms)

Today we cover 7.1 vocabulary for probability, and 7.2 some probability

7.1: Vocabulary for Probability

- Our last chapter is on probability.
- Probability is similar to counting
- 7.1 covers vocabulary for understanding the difference

- Life is uncertain, every snowflake is different
- In the aggregate, life is more certain
- If you flip a coin once, it will be heads or tails, but who knows which?
- If you flip a coin 1000 times, it will be heads between 450 and 550 times (with a 99.9% probability).

7.1: Experiments

- Reality is mysterious and wonderful

  It is worth observing.

- Some things you observe are unique: a sunset, a cloud

- Some things you observe are quite reproducible: when you flip a coin it lands on heads or tails, and each happens about 50% of the time

- An **experiment** is a planned observation of life whose goal is (usually) to confirm a reproducible result

- For example, we might plan an experiment where we flip 10 coins and count how many heads show up.

7.1: Sample spaces

- Our understanding of life is shaped by the constructs we place upon it
- Our understanding of coin flipping uses the construct of "heads" and "tails" to divide all of life's mysteries into two possible outcomes
- A **sample space** is a list of all the possible outcomes of an experiment
- If we pull one card from the deck, then our sample space can be the set of all 52 cards in the deck.
- If we draw five cards from the deck and don't care about order, then there are $\frac{52}{5}\frac{51}{4}\frac{50}{3}\frac{49}{2}\frac{48}{1} = 2,598,960$ possible outcomes

7.1: Events

- Many people rush through life and miss the details
- Suppose the experiment was flipping a single coin three times
- A reasonable sample space is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
- However some people might divide this up into "more heads than tails" and "more tails than heads"
- Each of these is an **event**, a subset of the sample space
- Mhtt $= \{HHH, HHT, HTH, THH\}$ has four sample points in it

7.1: Mutually exclusive

- You cannot both have more heads than tails and more tails than heads. If you had a tie, then neither was true!
- Two events are **mutually exclusive** if their intersection is empty; that is, it is not possible for both to happen at the same time.
- Not all events are mutually exclusive.
- For instance the event "get a head on the very first try!" is $\{HHH, HHT, HTH, HTT\}$ and so the intersection with "more heads than tails" is $\{HHH, HHT, HTH\}$
- There is an overlap, so we'll have to be careful

7.1: Experiment overview

1. Informally describe the **experiment**
2. Setup the sample space; decide the possible **outcomes**
3. Gather possible outcomes into interesting **events**
4. (Next section) describe how often an event is likely to occur if the experiment is repeated many times. This is the **probability**.
5. (STA291) After actually running the experiment, decide whether your probability calculation reflects reality
6. (STAxxx) Decide how many times to run the experiment before you can decide whether your probability calculation reflected reality

7.1: Summary

- We learned the words **experiment**, **sample space**, **event**, and **mutually exclusive**
- HW 7A is two questions. Easy questions.
- HW 7B and 7C are pretty similar to HW 6ABC
- Monday we will cover 7.2: Probability
- Depending on time we might cover it today

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- The event "rolling saved us money" is all those pairs that total to more than 6.
- There are 21 such pairs, and if all pairs are equally likely (the dice are fair), then that is $\frac{21}{36} = \frac{7}{12} \approx 58\%$

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- 16 ways to win, 32 ways total, so $\frac{16}{32} = \frac{1}{2} = 50\%$ chance
- Explicitly:
  HHHHH, HHHHT, HHHTH, HHHTT, HHTTT, HTHHH, HTTTH, HTTTT, THHHH, THHHT, THTTT, TTHHH, TTTHH, TTTHT, TTTTH, TTTTT

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- It should be the same for getting an odd number of tails, right? Tails, heads, what is the difference?
- But you either get an odd number of heads, or an odd number of tails, and not both, so each should be about equally likely: 50%

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- However, that's not very likely to happen and quite expensive to plan for.
- If each bulb is independent, that is $(0.1\%)^{700} \approx 0\%$ chance of this happening

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- The odds of none going out is $(99.9\%)^{700} \approx 50\%$, the odds of one are $700 \cdot (0.1\%)(99.9\%)^{699} \approx 35\%$
- Total is: $0.844 = 84.4\%$ chance that at most one breaks, so not too bad. Every 6 weeks you'll have a light out and no replacement, but not too bad.

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- What are the odds that 10 is enough?
- The odds of none going out is $(99.9\%)^{7000} \approx 0.1\%$,
  exactly one are $7000 \cdot (0.1\%)(99.9\%)^{6999} \approx 0.6\%$,
  exactly two are $\frac{7000 \cdot 6999}{2} \cdot (0.1\%)^2(99.9\%)^{6998} \approx 2.2\%$,
  ...

0	1	2	3	4	5	6	7	8	9	10
0.1	0.6	2.2	5.2	9.1	12.7	14.9	14.9	13.0	10.1	7.0

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- Total is: $0.902 = 90.2\%$ chance that at most ten break, so really we're even more certain to be ok now; every 10 weeks we'll be short a bulb.

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- The larger the population, the less extreme the whims of fortune
- This is why insurance is important; the risk to one person is great
- The risk to 10,000 people is quite small, much less than 10,000 times the risk of one

Round table

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- Sample space is:
  ABCD, ABCE, ABDC, ABDE, ABEC, ABED, ACBD, ACBE, ACDB, ACDE, ACEB, ACED, ADBC, ADBE, ADCB, ADCE, ADEB, ADEC, AEBC, AEBD, AECB, AECD, AEDB, AEDC, BCDE, BCED, BDCE, BDEC, BECD, BEDC

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- The event is all those with DE or ED (be careful of wraparound)
- 12 bad out of 30 total is 40% chance for showers (of fists)